ECE-161C
Color

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( _with thanks to David Forsyth_ )
Color

- so far we have talked about geometry
  - where is a 3D point map mapped into, in terms of image coordinates?
  - perspective projection
    \[(x, y) = \frac{f}{Z} (X, Y)\]

- and light
  - what is the brightness of that image pixel
  - radiance of Lambertian surfaces, point source at infinity
    \[L = \frac{\rho_d}{\pi} E \cos \langle n, s \rangle\]

- today we look at the missing link: color
Radiometry for color

- All definitions are now “per unit wavelength”
- All units are now “per unit wavelength”
- All terms are now “spectral”
- Radiance becomes spectral radiance $L^\lambda(x, \theta, \phi)$
  - watts per square meter per steradian per unit wavelength
  - radiance emitted in $[\lambda, \lambda+d\lambda]$ is
    $$L^\lambda(x, \theta, \phi)d\lambda$$
- Spectral radiosity, spectral exitance, etc.
- Spectral BRDF
  $$\rho_{bd}^\lambda(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{L_o^\lambda(P, \theta_o, \phi_o)}{L_i^\lambda(P, \theta_i, \phi_i)\cos \theta_i d\omega}$$
Spectral albedos

different leaves, color names attached. Different colors typically have different spectral albedo, but that different spectral albedoes may result in the same perceived color (compare the two whites). Measurements by E.Koivisto.
The appearance of colors

- To talk about color we need to know how it is perceived by people.

- Hering, Helmholtz: color appearance strongly affected by nearby colors, adaptation to previous views, “state of mind”.

- Film color mode: view colored surface through a hole in a sheet, so that the color looks like a film in space; controls for nearby colors, and state of mind.

- Principle of trichromacy: it is possible to match almost all colors, viewed in film mode, using only three primary sources.

- Most of what follows discusses film mode.
Color matching experiments

- Subject shown a split field:
  - one side shows the light whose color one wants to measure,
  - other a weighted mixture of primaries (fixed lights).

- Subject adjusts dials so as to make mixture equal to test.
Color matching experiments (cont’d)

- Most colors can be represented as a mixture of \( P_1, P_2, P_3 \)
  \[ M = a \ P_1 + b \ P_2 + c \ P_3 \]
  where the = sign should be read as “matches”
- This is additive matching.
- Important because if two people who agree on \( P_1, P_2, P_3 \) need only supply \((a, b, c)\) to describe a color.
- Some colors can’t be matched like this: instead we need
  \[ M + a \ P_1 = b \ P_2 + c \ P_3 \]
- This is subtractive matching.
- We can interpret it as \((-a, b, c)\)
The principle of trichromacy

- Experimental facts:

- Three primaries will work for most people if we allow subtractive matching
  - Exceptional people can match with two or only one primary.
  - This could be caused by a variety of deficiencies.

- Most people make the same matches.
  - There are some anomalous trichromats, who use three primaries but make different combinations to match.

- Color matching is linear (Grassman’s laws)
Grassman’s Laws

1. mixture of coordinates matches the mixture of the lights

\[ T_1 = a_1 P_1 + b_1 P_2 + c_1 P_3 \]
\[ T_2 = a_2 P_1 + b_2 P_2 + c_2 P_3 \]
\[ T_1 + T_2 = (a_1 + a_2) P_1 + (b_1 + b_2) P_2 + (c_1 + c_2) P_3 \]

2. equal coordinates means equal lights

\[ T_1 = aP_1 + bP_2 + cP_3 \]
\[ T_2 = aP_1 + bP_2 + cP_3 \]
\[ T_1 = T_2 \]

3. matching is linear

\[ T_1 = aP_1 + bP_2 + cP_3 \iff kT_1 = kaP_1 + kbP_2 + kcP_3 \]

► note: these statements are as true as any biological law.
Linear color spaces

- Because color matching is linear in the primaries it makes sense to think of the **primaries as the basis of a linear color space**

- **Note that the space is infinite dimensional**
  1. think of $L(\lambda)$ as $L(\lambda_1, \lambda_2, \ldots, \lambda_N)$
  2. take $N$ to infinity

- **The space of valid colors is a 3D-subspace**

- **Problem: the basis is not necessarily orthogonal**
Color matching functions

How do we get the color coordinates?

- pick a source $\delta(\lambda - \lambda_0)$ of unit radiance at wavelength $\lambda$
- we know that there is a set of weights that matches it
- denote the weight of $P_i$ by $f_i(\lambda_0)$

\[
\delta(\lambda - \lambda_0) = \sum_i f_i(\lambda_0) P_i
\]

- repeat for all $\lambda_0$

The $f_i(\lambda)$ are called color matching functions because we can write

\[
L(\lambda) = \int L(\lambda_0) \delta(\lambda_0 - \lambda) d\lambda_0 = \sum_i \left\{ \int L(\lambda_0) f_i(\lambda_0) d\lambda_0 \right\} P_i = \sum_i \omega_i P_i
\]
Color matching functions (cont’d)

- The color coordinates are the projections of the spectral radiance on the three matching functions

\[ a = \int L(\lambda_0) f_1(\lambda_0) d\lambda_0 \]
\[ b = \int L(\lambda_0) f_2(\lambda_0) d\lambda_0 \]
\[ c = \int L(\lambda_0) f_3(\lambda_0) d\lambda_0 \]

- Note that this means that we can specify the color space by specifying a set of matching functions

- This can sometimes lead to primaries that are not physically feasible (e.g. if we constrain the MFs to be non-negative)

- OK because we really only care about the coordinates
Linear color spaces

- **RGB**: primaries are monochromatic energies are 645.2nm, 526.3nm, 444.4nm.
- Color matching functions have negative parts.
- Some colors can be matched only subtractively.
RGB space

- primaries are the phosphors monitors use as primaries
- the color space is a cube
- axes represent the amount of red, green, and blue
- each color has coordinates between 0 and 1 (0 and 255)
Linear color spaces

**CIE XYZ:** color matching functions positive

due to this *primaries are imaginary*, but have convenient properties

color coordinates are (X,Y,Z), where X is amount of X primary, etc.
CIE xy

- 2D is easier to visualize than 3D
- Usually work with CIE xy, where
  - \( x = \frac{X}{X+Y+Z} \)
  - \( y = \frac{Y}{X+Y+Z} \)
- This is the intersection of the color space with the plane \( X+Y+Z=1 \)
Color perception

- human perception of color is best described in terms of three fundamental color properties
- **hue**: this is what you would refer to as the color itself

  e.g. red vs yellow

- note that there is a **circular structure** (we start and finish with red)
- for this reason color is usually visualized in a **color wheel**
Color perception

- **saturation**: this is the adjective: e.g. “vivid” red, “pale” yellow
- it is the radial coordinate on the color wheel
- colors at the center are unsaturated, colors at the boundaries are highly saturated
Color perception

- **intensity**: is the amount of brightness
- “dark” yellow vs “light” yellow
- the color wheel can be replicated at each intensity level

**Note that:**
- the center is always the intensity level
- at zero intensity, hue and saturation are irrelevant
- saturation becomes more important at higher intensities
Perceptual color spaces

**HSV:** hue, saturation, value (intensity) is a natural “perceptual” color space

- very non-linear, colors form a cone
- this can be bad for some applications
- e.g. TVs where primaries have nothing to do with perception but with building CRTs
- but very good for others
Perceptual color spaces

- note that, in general, we can extract the three properties from any space
- this picture shows H, S, and V in CIE xy
- the problem is that the transformation is non-linear
- in CIExy two colors that are near do not necessarily “look” similar
MacAdam ellipses

- Ellipses represent colors that human observers matched to color on their center; boundary shows just noticeable difference.
- Left: magnified 10x. The ellipses at the top are larger than those at the bottom, and rotate as they move up.
- Difference in x,y coordinates is poor guide to color difference!
Perceptual color spaces

- **Uniform**: equal (small!) steps give the same perceived color changes

- This is important, for example for image retrieval based on color similarity

- Compute an histogram of color values for each image and measure image similarity by the similarity of the color histograms

- Works surprisingly well (more on this later)
Uniform color spaces

- but there are many other applications
- lots of research on constructing color spaces so that differences in coordinates are a good guide to differences in color
- e.g. CIE u’v’ which is a projective transform of x, y
- transform x,y so that ellipses are most like one another. Figure shows the transformed ellipses
Uniform color spaces

- **L*a*b** is quite popular
- 3rd order approximation to the Munsell color notation system (which we will not get into)
- represents colors relative to a reference white point, that defines white light
- the whitest light generated by a given device. (In this sense L*a*b* is device dependent; equal colors are truly equal iff relative to the same white point.)
TV color spaces

- in between linear and perceptual
  - decomposition into intensity plus two color channels (e.g. R-Y, B-Y)
  - linear transformation of RGB

- compatibility: modulate color on top of BW; MPEG support of existing TV sets

- compression: main perceptually relevant feature is that people have less color than intensity resolution

- NTSC: YIQ (1/2 color bandwidth)

- JPEG/MPEG: YCbCr (4:2:0 for broadcast, 4:2:2 for studio)
Printers

- for printers the story is quite different:
  - we have to consider **subtractive** color spaces
  - ink pigments **remove** color from incident light
  - remaining light is reflected by the paper
  - e.g. red ink is a die that absorbs blue and green

- most common space is **CMY** with primaries
  - cyan (white minus red)
  - magenta (white minus green)
  - yellow (white minus blue)

- but mixtures can still be evaluated wrt to the RGB space
  - E.g. \( C+M = (W-R) + (W-G) = R+G+B-R-G = B \)
  - \((W+W = W\) because ink cannot increase the amount of light)
CMY and CMYK

- the relationship to RGB can be summarized pictorially

- it turns that it is difficult to generate black in this way
- lots of synchronization problems
- CMYK solves this by explicitly including a black ink
Conversions

- in practice, we often have to do color-space conversions

- here are some of the formulas (for more details see http://www.easyrgb.com/math.html)
  - RGB to XYZ: see homework
  - CIE XYZ to CIE u'v'
    \[
    (u', v') = \left( \frac{4X}{X + 15Y + 3Z}, \frac{9Y}{X + 15Y + 3Z} \right)
    \]
  - CIE XYZ to L*a*b
    \[
    L^* = 116 \left( \frac{Y}{Y_n} \right)^{\frac{1}{3}} - 16 \quad a^* = 500 \left[ \left( \frac{X}{X_n} \right)^{\frac{1}{3}} - \left( \frac{Y}{Y_n} \right)^{\frac{1}{3}} \right] \quad b^* = 200 \left[ \left( \frac{Y}{Y_n} \right)^{\frac{1}{3}} - \left( \frac{Z}{Z_n} \right)^{\frac{1}{3}} \right]
    \]
  
  \((X_n, Y_n, Z_n)\) are \((X, Y, Z)\) coordinates of a reference white patch.
Conversions

• RGB to CMY

\[ C = 1-R \quad M = 1-G \quad Y=1-B \]

• CMY to CMYK

\[ K = \min(C,M,Y) \quad C = C-K \quad M = M-K \quad Y=Y-K \]

• RGB to YCrCb

\[ Y = 0.29900R + 0.58700G + 0.11400B \]
\[ C_b = -0.16874R - 0.33126G + 0.50000B \]
\[ C_r = 0.50000R - 0.41869G - 0.08131B \]
Conversions

- **RGB to HSV**

\[
\begin{align*}
\text{Min} &= \min(R,G,B); \quad \text{Max} = \max(R,G,B); \\
\Delta &= \text{Max} - \text{Min}; \quad V = \text{Max}; \\
\text{if} \ ( \Delta == 0 \) \ {H} &= 0; \quad S = 0 \\
\text{else} \ {S} &= \Delta / \text{Max}; \\
\Delta R &= \left[ (\text{Max} - R) / 6 \ + \ (\Delta / 2) \right] / \Delta; \\
\Delta G &= \left[ (\text{Max} - G) / 6 \ + \ (\Delta / 2) \right] / \Delta \\
\Delta B &= \left[ (\text{Max} - B) / 6 \ + \ (\Delta / 2) \right] / \Delta \\
\text{if} \ ( R == \text{Max} ) \ {H} &= \Delta B - \Delta G \\
\text{else if} \ ( G == \text{Max} ) \ {H} &= (1 / 3) + \Delta R - \Delta B \\
\text{else if} \ ( B == \text{Max} ) \ {H} &= (2 / 3) + \Delta G - \Delta R \\
\text{if} \ ( H < 0 ) \ ; \ H += 1; \text{if} \ ( H > 1 ) \ ; \ H -= 1 \\
\end{align*}
\]
Why three primaries?

- color perception is the result of evolution
- let’s look at the human eye
- Two types of receptors:
  - rods: responsible for vision at low light levels, do not mediate color vision
  - cones: active at higher light levels, capable of color vision
- rods dominate in the periphery of the eye,
- the center is mostly composed of cones
- three types of cones: S, M, and L
Rod vs cone density

- very high concentration of cones in the center
- high resolution and color
- density decays very quickly
Color receptors and trichromacy

- **Trichromacy** is justified: three types of cones, which vary in their sensitivity to light at different wavelengths.

- Picture shows a **cone mosaic**, spatial distribution of the different types of cones.

- Blue: S cones
  - Red: L cones
  - Green: M cones

- Note the different densities.
Color receptors

- relative sensitivity as a function of wavelength. S (for short) cone responds most strongly at short wavelengths; the M (for medium) at medium wavelengths and the L (for long) at long wavelengths.
- occasionally called B, G and R cones respectively, but that’s misleading - you don’t see red because your R cone is activated.
- explains 3 primaries
Any questions?