ECE-161C
Cameras

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Image formation

- all image understanding starts with understanding of image formation:
  - projection of a scene from 3D world into image on 2D plane

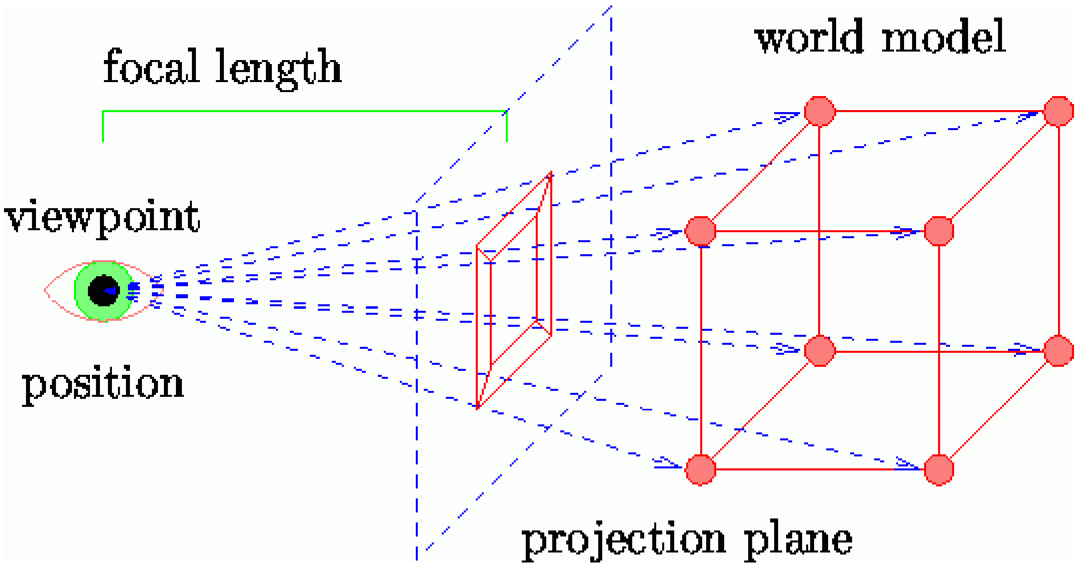
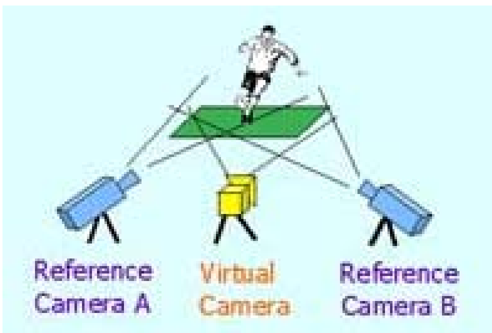


Image formation

- first of all, why do we care about this?
- 1) allows us to create (“render”) imaginary scenes
  - special effects, games, architecture/visualization, etc.
  - build a CAD model of the scene and then render from different views

Image formation

- e.g. a "fly-through" camera that allows you to see a sports event from new angles



- we know
  - where the camera is
  - knowing the projection equations allows us to recreate the image from a 3D model of the scene
  - computer graphics is mostly about this

Image formation

- 2) even better, we could reconstruct the 3D model from images
- rendering:
  - 3D world to 2D image
- scene reconstruction:
  - 2D images to 3D model
  - this turns out to be much harder because a 2D projection is consistent with many 3D scenes
  - one image is usually not enough, but can be done from a collection of images

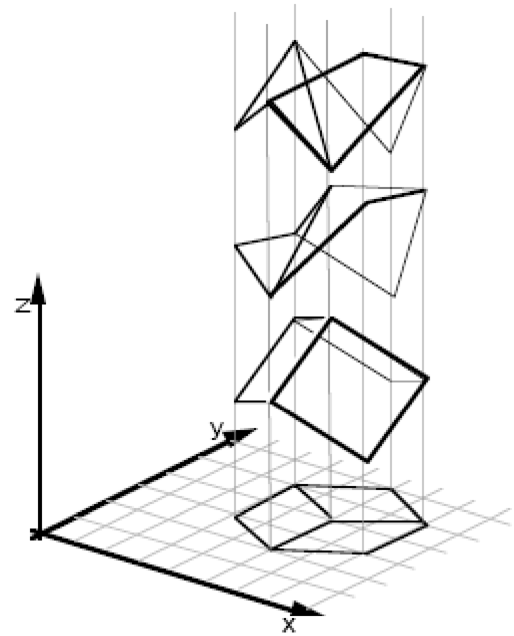


Image formation

- when multiple images are available, it is possible to
  - register them
  - deduce the mapping from 2D to 3D
  - extract a 3D model of the scene
  - render from different angles

Image formation

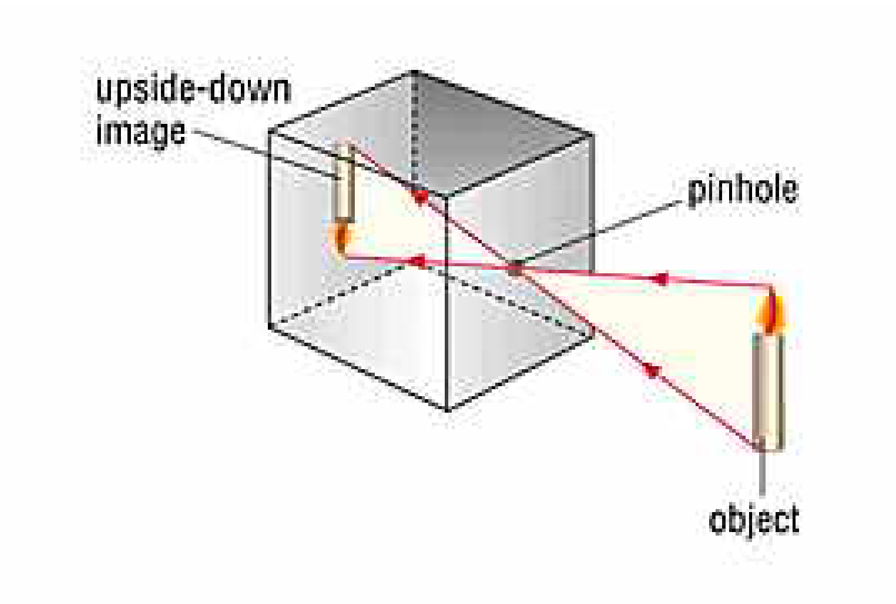
- even when we do not care about the 3D scene per se
  - knowing the geometry is important for many tasks
  - note the appearence changes as the cars move
  - this is due to perspective
  - has to be accounted for even though the goal is tracking not reconstruction

Image formation

- even for rigid scenes that do not change that much
  - a change of perspective will create massive pixel changes
  - to compensate for this, one has to understand the projection equations
- this turns out to be quite complicated
  - as usual in science, we simplify as much as we can
  - for example, we adopt the pinhole camera model

Pinhole camera

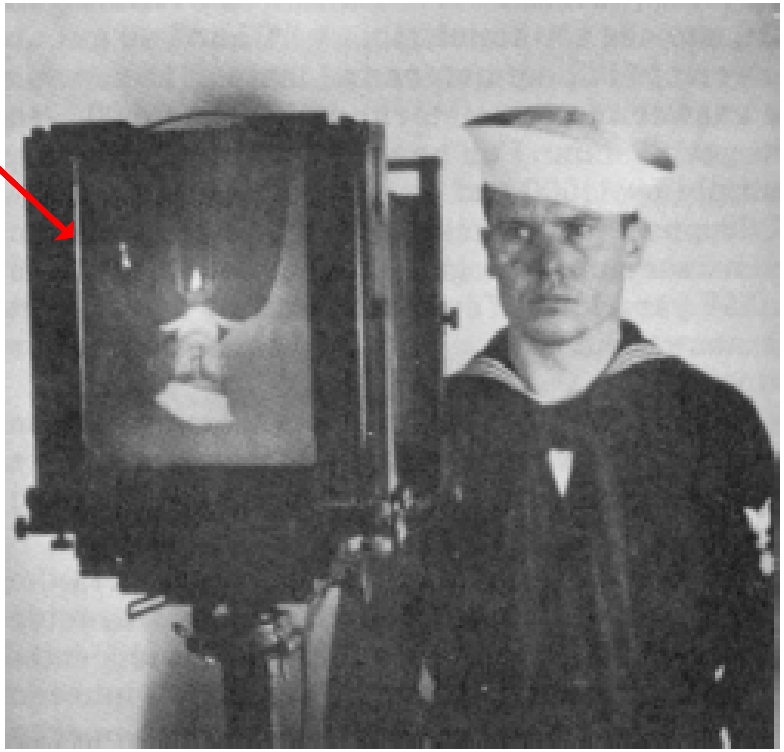
- we assume that a camera is
  - a black box
  - with an infinitesimally small hole on one face
  - the hole is so small that only one ray of light passes through it and hits the other side



real pinhole camera made by Kodak for schools, circa 1930

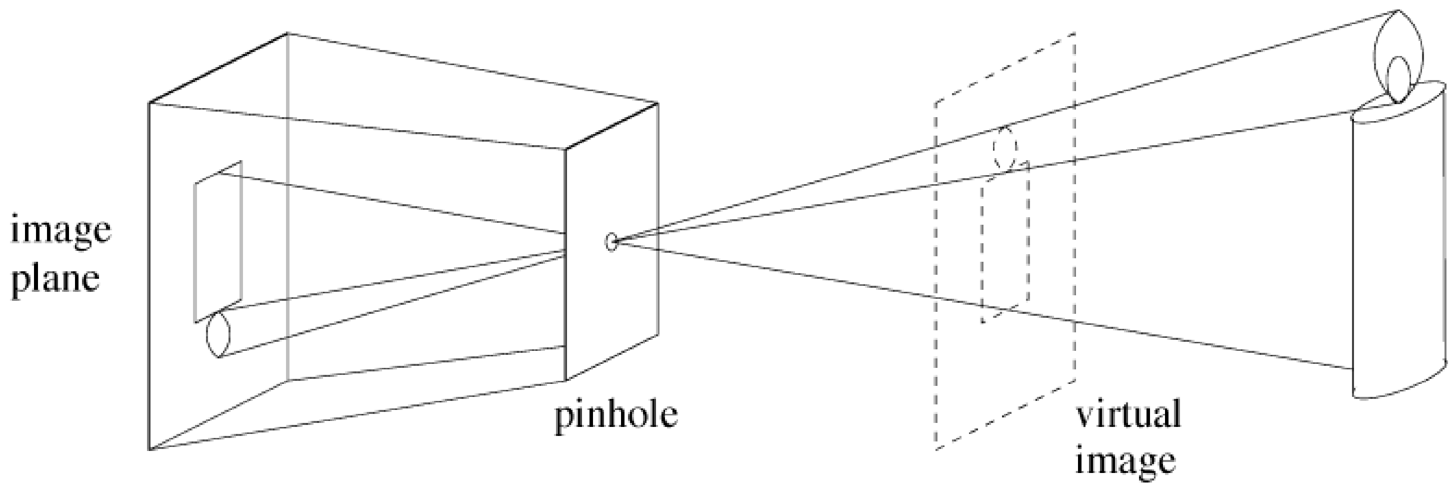
Pinhole camera

- by placing photo-sensitive material in the back wall you will get an upside-down replica of the scene
  - this is the image plane
  - to avoid the mathematical inconvenience of this inversion
  - we consider a plane outside of the camera
  - this is called the virtual image plane



Pinhole camera

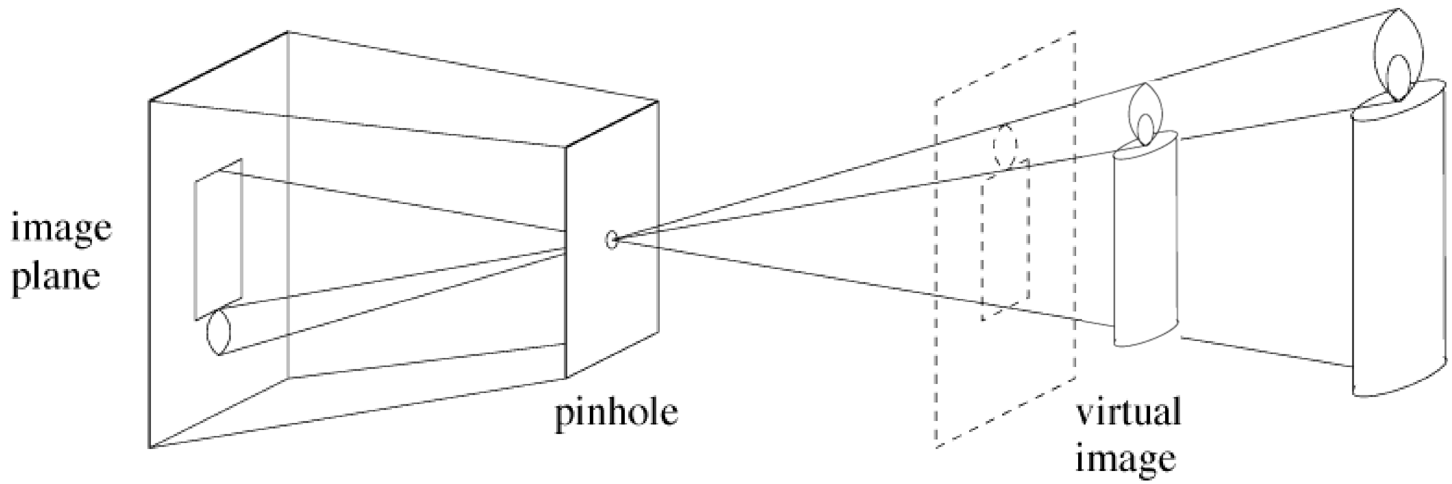
- the virtual image plane



- it is an abstraction
- exactly the same as the image plane, with the exception that there is no inversion

Pinhole camera

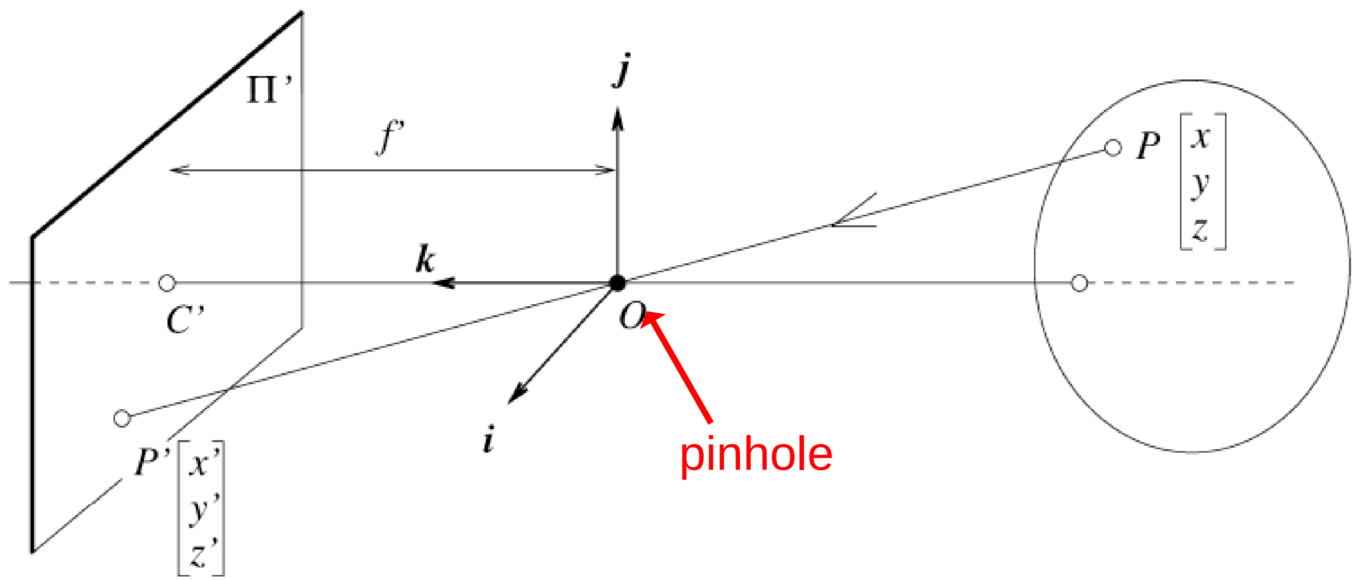
- one important property:
  - objects that are far away become smaller in the image plane



  - we suspect that distance to the camera plays an important role in perspective projection
  - in particular we would expect image size proportional to 1/d

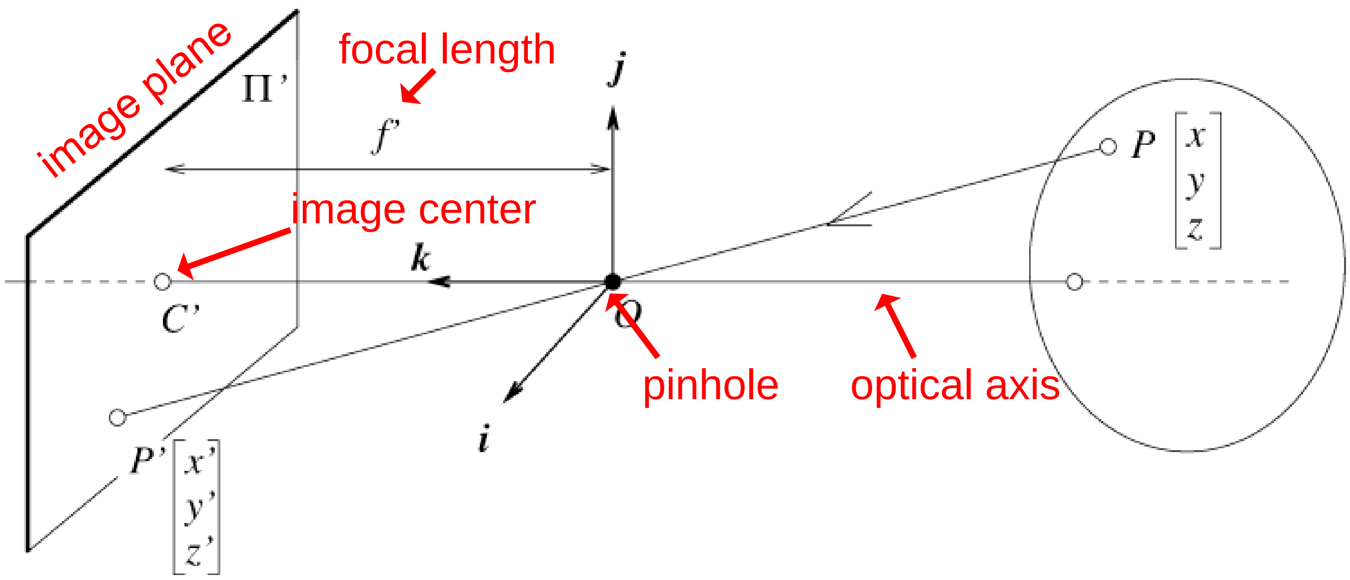
Coordinates

- to relate world point *P* to image point *P'*
  - we need a coordinate system
  - the 1/d dependence suggests using pinhole as origin
  - we also make two coordinate axes ***(i,j)*** a basis of the image plane and the third ***(k)*** orthogonal to it (measures depth)



Pinhole camera

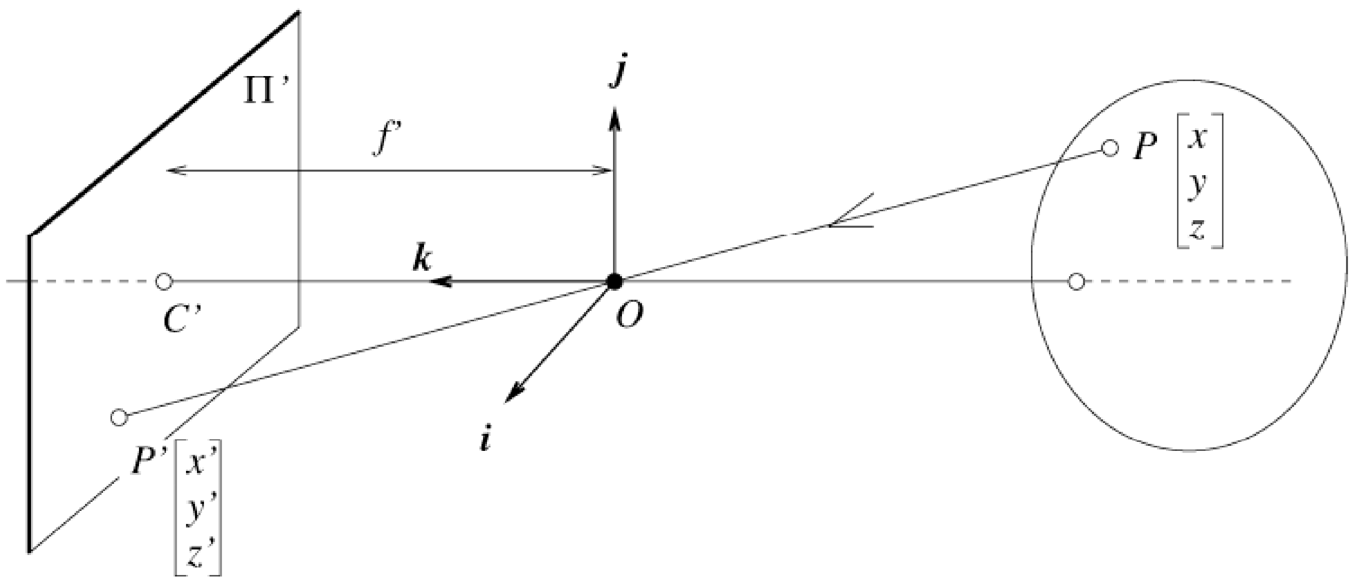
- definitions:
  - line perpendicular to image plane, through pinhole, is the optical axis
  - point where optical axis intersects image plane is the image center
  - distance f between image plane and pinhole is the focal length



Projection equations

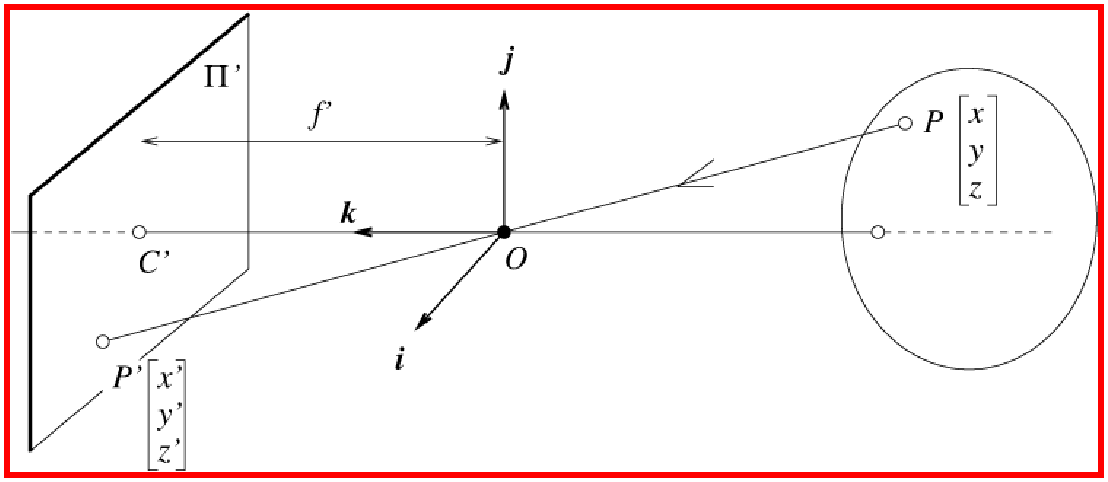
- note that
  - P, O, P' are on the same line
  - this implies that there is a λ such that *OP' =* λ *OP*, and

$$\begin{cases} x' = \lambda x \\ y' = \lambda y \\ f = \lambda z \end{cases} \Rightarrow \lambda = \frac{f}{z} \Rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = f \begin{pmatrix} x/z \\ y/z \end{pmatrix}$$



Perspective projection

- this is the basic equation of perspective projection

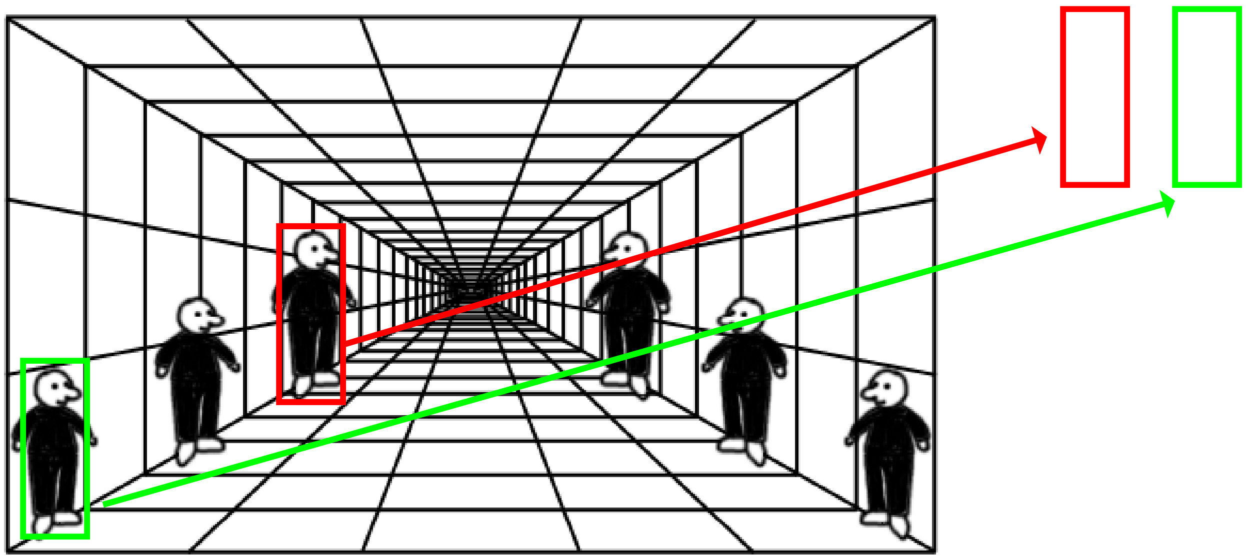


$$\begin{pmatrix} x' \\ y' \end{pmatrix} = f \begin{pmatrix} x/z \\ y/z \end{pmatrix}$$

- note that
  - there is indeed an inverse dependence on the depth Z
  - far objects become small

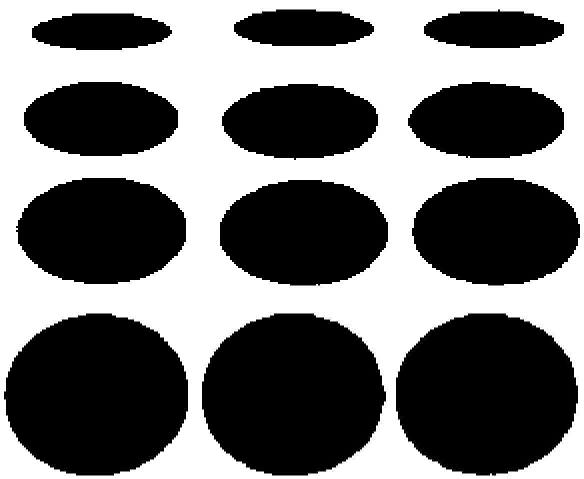
Perspective projection

- this is a very powerful cue for scene understanding
  - and fun too!
  - note that the visual system infers all sorts of properties from perspective cues
  - e.g. size

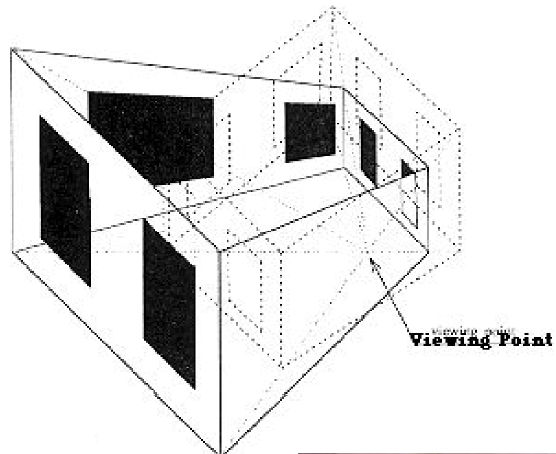


Perspective projection

- or shape



or proximity



Perspective projection

- is conceptually very simple

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = f \begin{pmatrix} x/z \\ y/z \end{pmatrix}$$

- but is highly non-linear and usually hard to work with
- e.g. assume you have a big plane on the scene, e.g. a wall

$$z = ax + by$$

- then

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = f \begin{pmatrix} x/(ax+by) \\ y/(ax+by) \end{pmatrix}$$

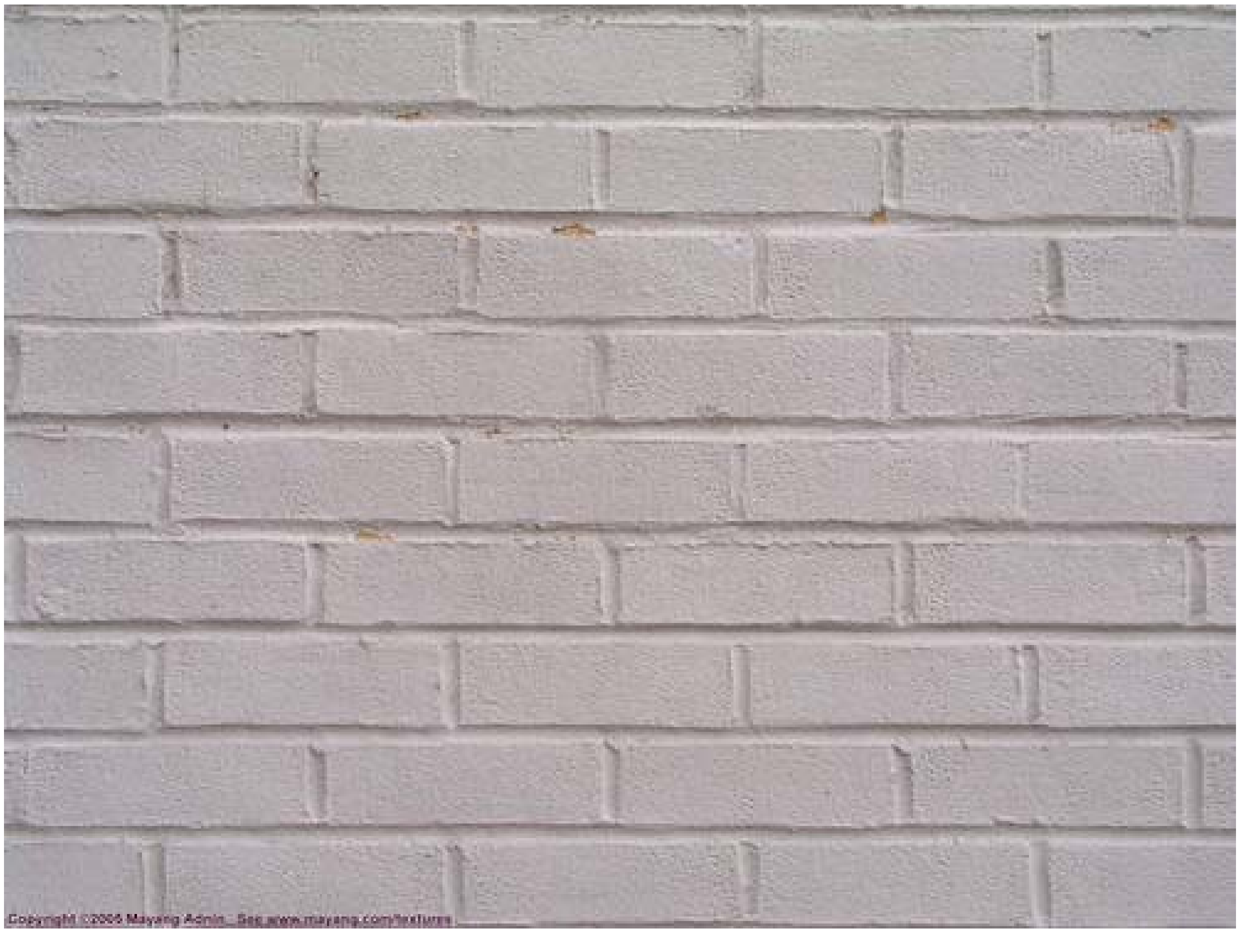
- the image coordinates depend highly non-linearly on the world coordinates

Prospective projection

- this is the reason why we see this

Prospective projection

- instead of this



Projective projection

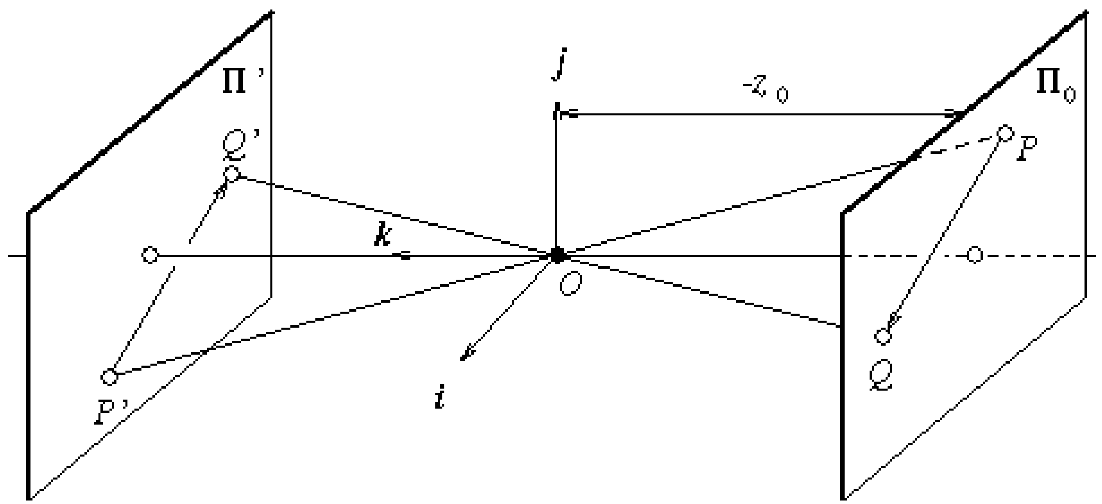
- since the size of the wall is constant
- far away (large z) distances appear to shrunk in the image
- in many cases, this non-linearity is too much to handle
- we look for approximations

Affine projection

- consider a plane parallel to the image plane
  - this plane has equation $z = C$ and the projection equation becomes

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \frac{f}{C} \begin{pmatrix} x \\ y \end{pmatrix} = m \begin{pmatrix} x \\ y \end{pmatrix}, \quad m = \frac{f}{C}$$

  - image coordinates are simply a re-scaling of the 3D coordinates



Affine projection

- scaling:
  - if m <1 image points are closer than 3D points,
  - else they are further away
  - this can be seen by noting that, for $P=(x_p,y_p)$, $Q=(x_q,y_q)$

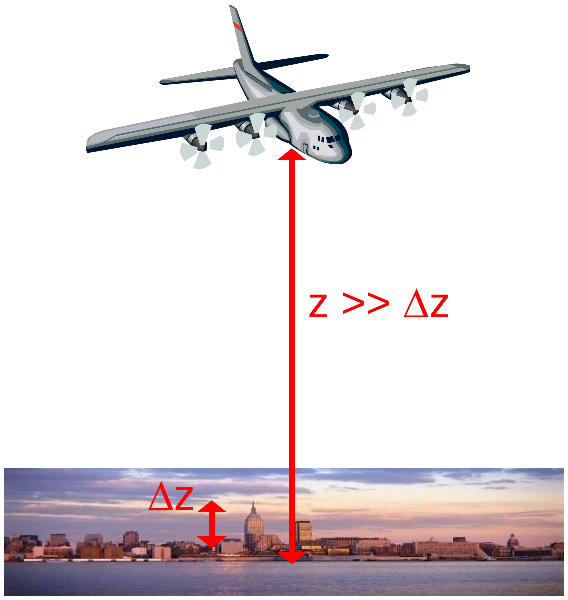
$$
\begin{aligned}
d(P',Q') &= \sqrt{\left(x'_p - x'_q\right)^2 + \left(y'_p - y'_q\right)^2} \\
&= \sqrt{m^2\left(x_p - x_q\right)^2 + m^2\left(y_p - y_q\right)^2} \\
&= |m| \; d(P,Q)
\end{aligned}
$$

  - this is also captured by the relation through a scaling matrix

$$
\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix}
$$

Affine projection

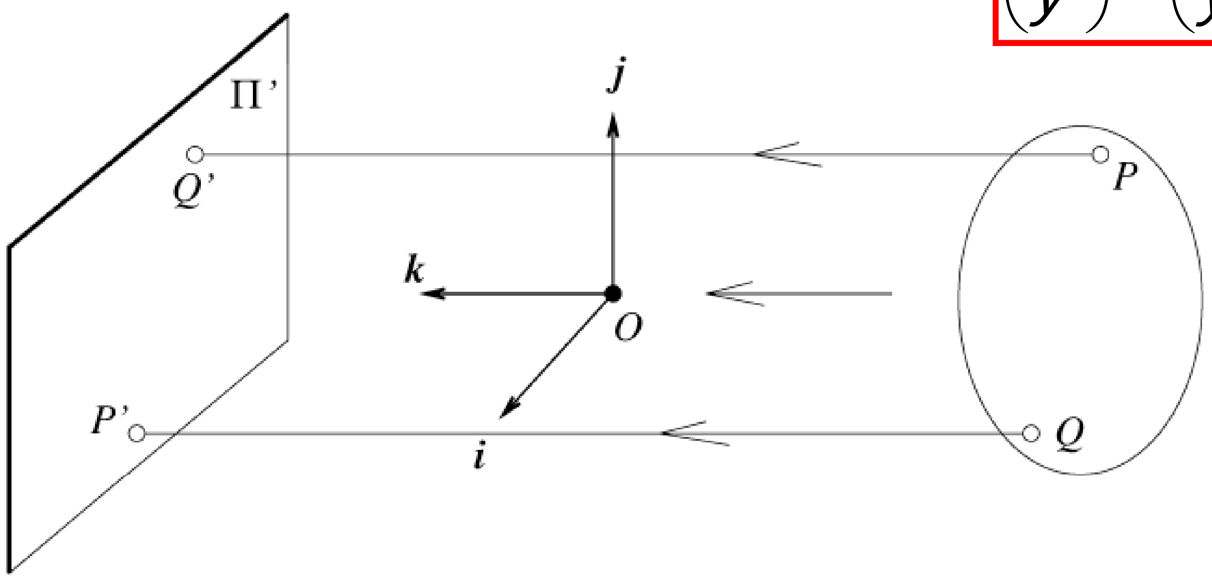
- when can we use this approximation?
  - we are assuming z constant
  - this is acceptable if the variation of depth in the scene is much smaller than the average depth
  - e.g. an airplane taking aerial photos



Orthographic projection

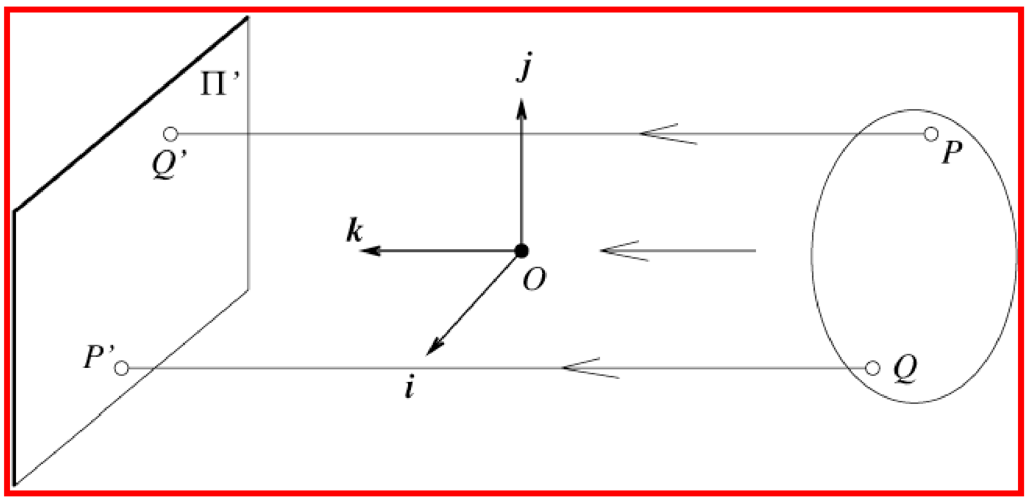
- if the camera is always at (approximately) the same distance from the scene:
  - m only contributes a change of scale that we do not care much about (e.g. measure in centimeters vs meters)
  - it is common to normalize to *m = 1*
  - this is orthographic projection

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$



Orthographic projection

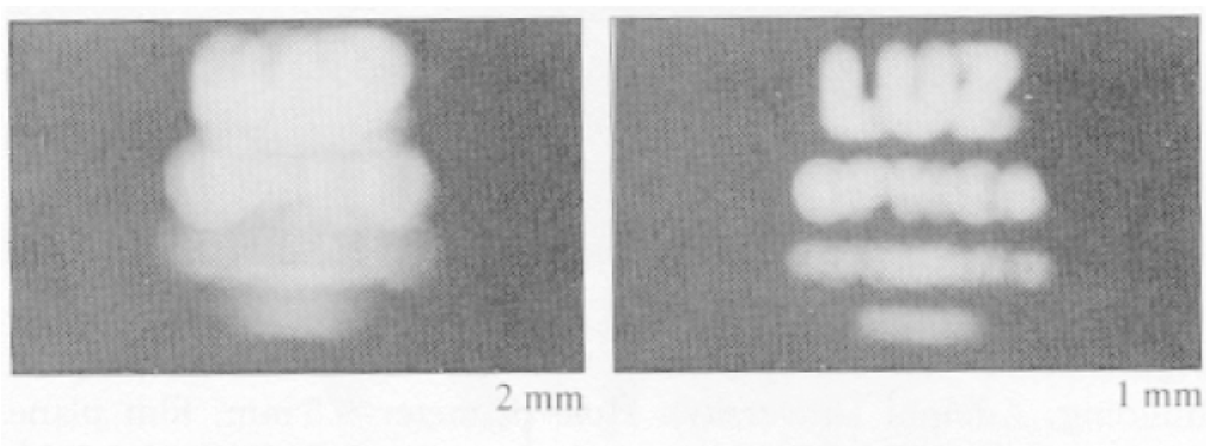
- this is, of course, very convenient:
  - "image coordinates = world coordinates"
  - there are not that many scenarios in which it is a good approximation
  - nevertheless can be a good model for a preliminary solution
  - which is then refined with a more complicated model



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

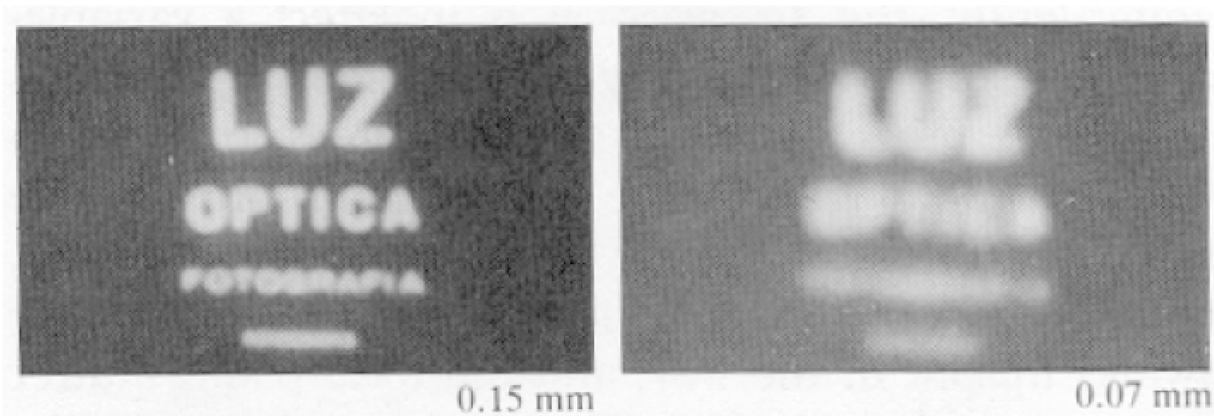
Lenses

- so far we have assumed the pinhole camera
- in practice we cannot really build such a camera and obtain decent quality
- problems:
  - when pinhole is too big
  - many directions are averaged, blurring the image



Lenses

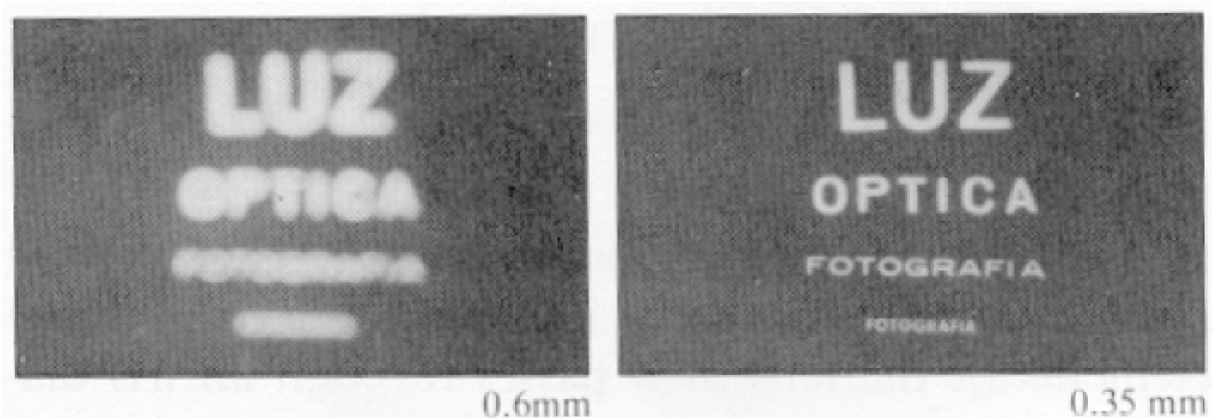
- pinhole problems:
  - if the pinhole is too small
  - we have diffraction effects which also blur the image



- there is a correct pinhole size from an image distortion point of view, but that introduces other problems

Lenses

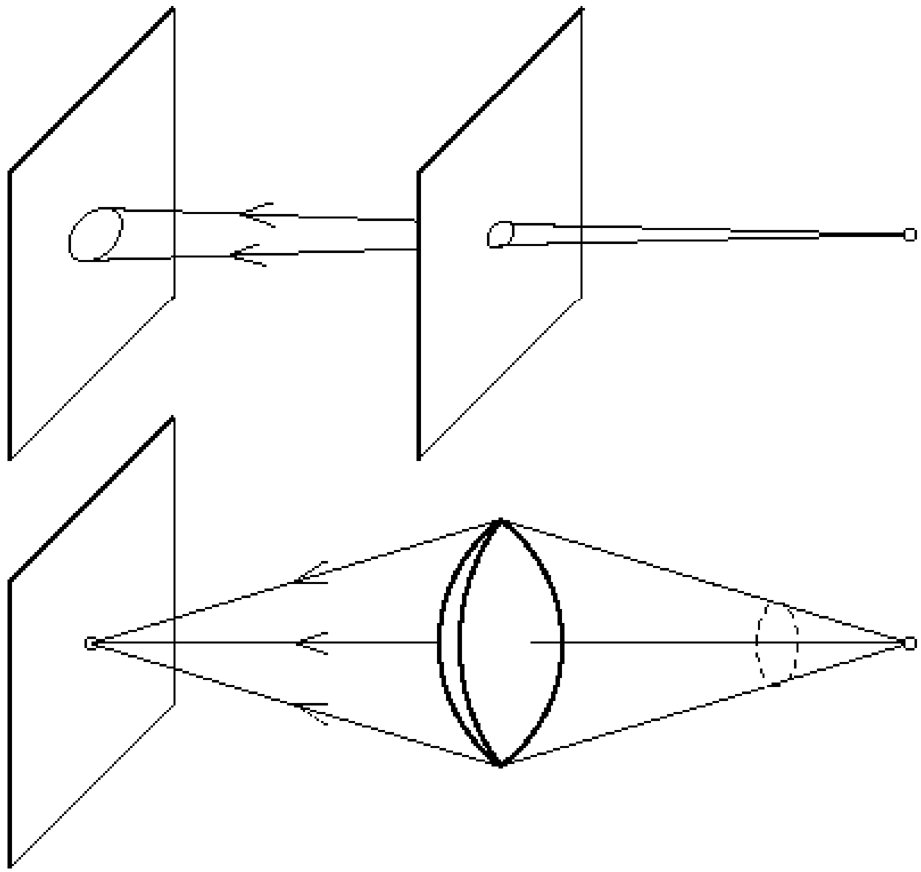
- pinhole problems:
  - for the “correct” pinhole size
  - we cannot get enough light in the camera to sufficiently excite the recording material
  - generally, pinhole cameras are *dark*,
  - a very small set of rays from a particular point hits the screen



  - this is the reason why we need camera lenses

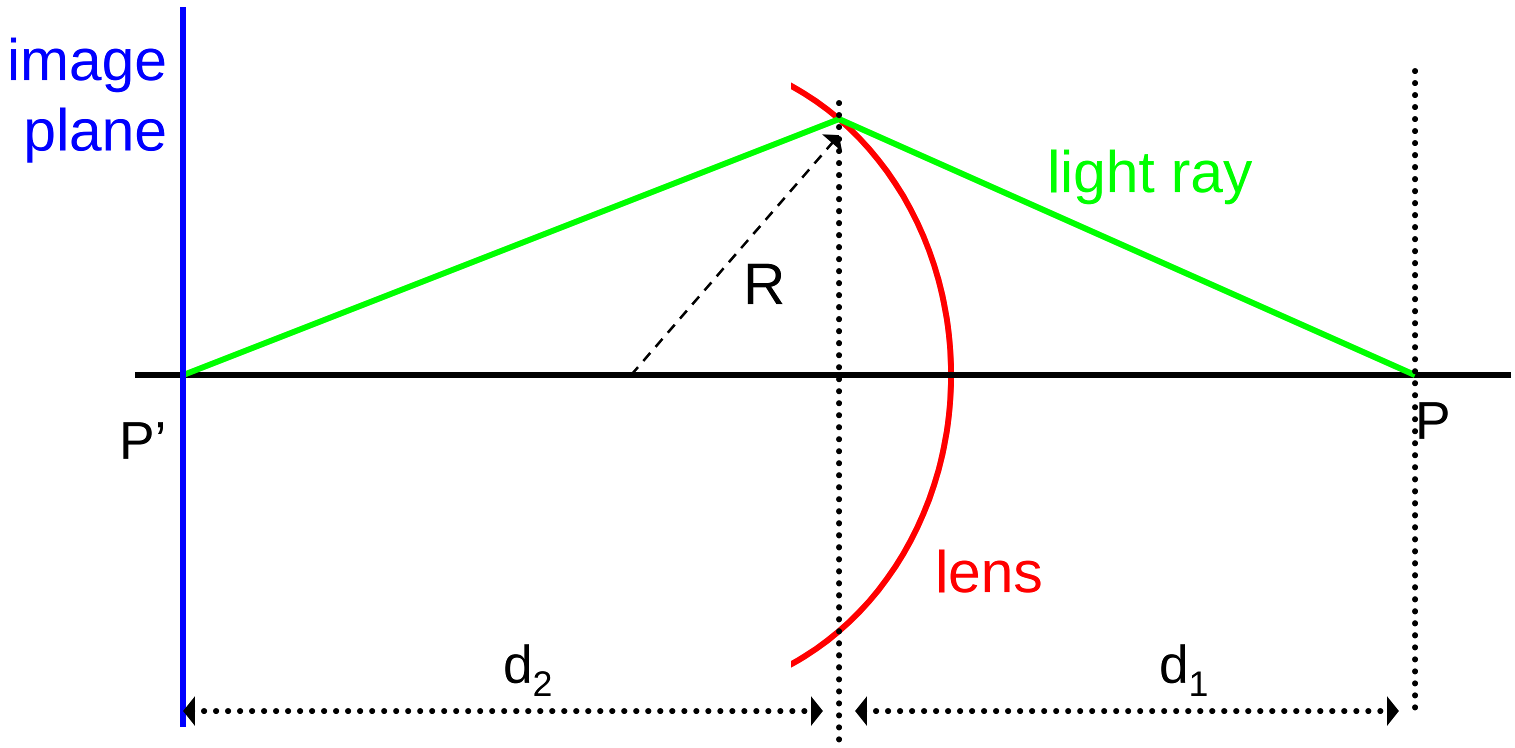
Lenses

- the basic idea is:
  - lets make the aperture bigger so that we can have many rays of light into the camera
  - to avoid blurring we need to concentrate all the rays that start in the same 3D point
  - so that they end up on the same image plane point



Lenses

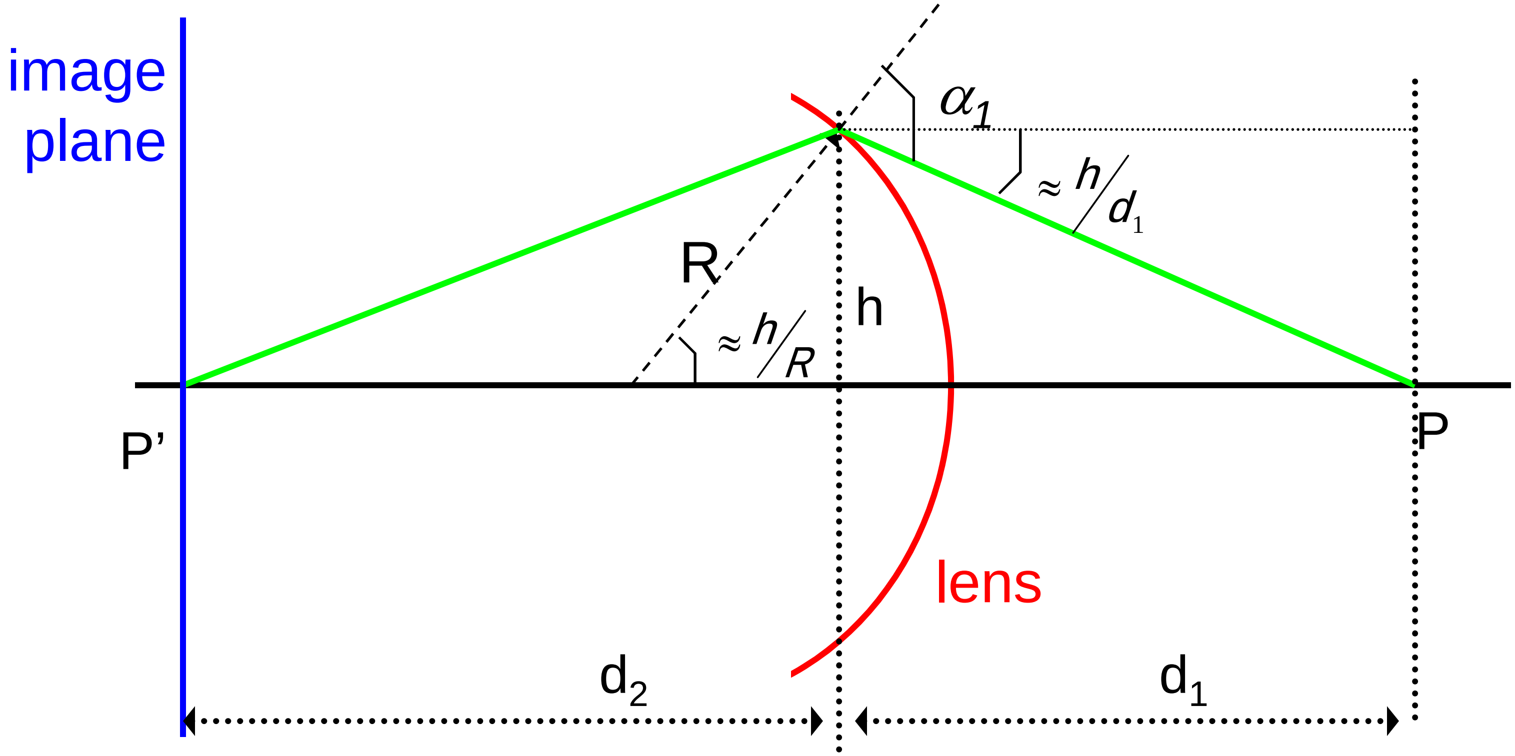
- the geometry is as follows



R: radius of curvature of the lens
d_1: distance from 3D point to lens
d_2: distance to image plane

Lenses

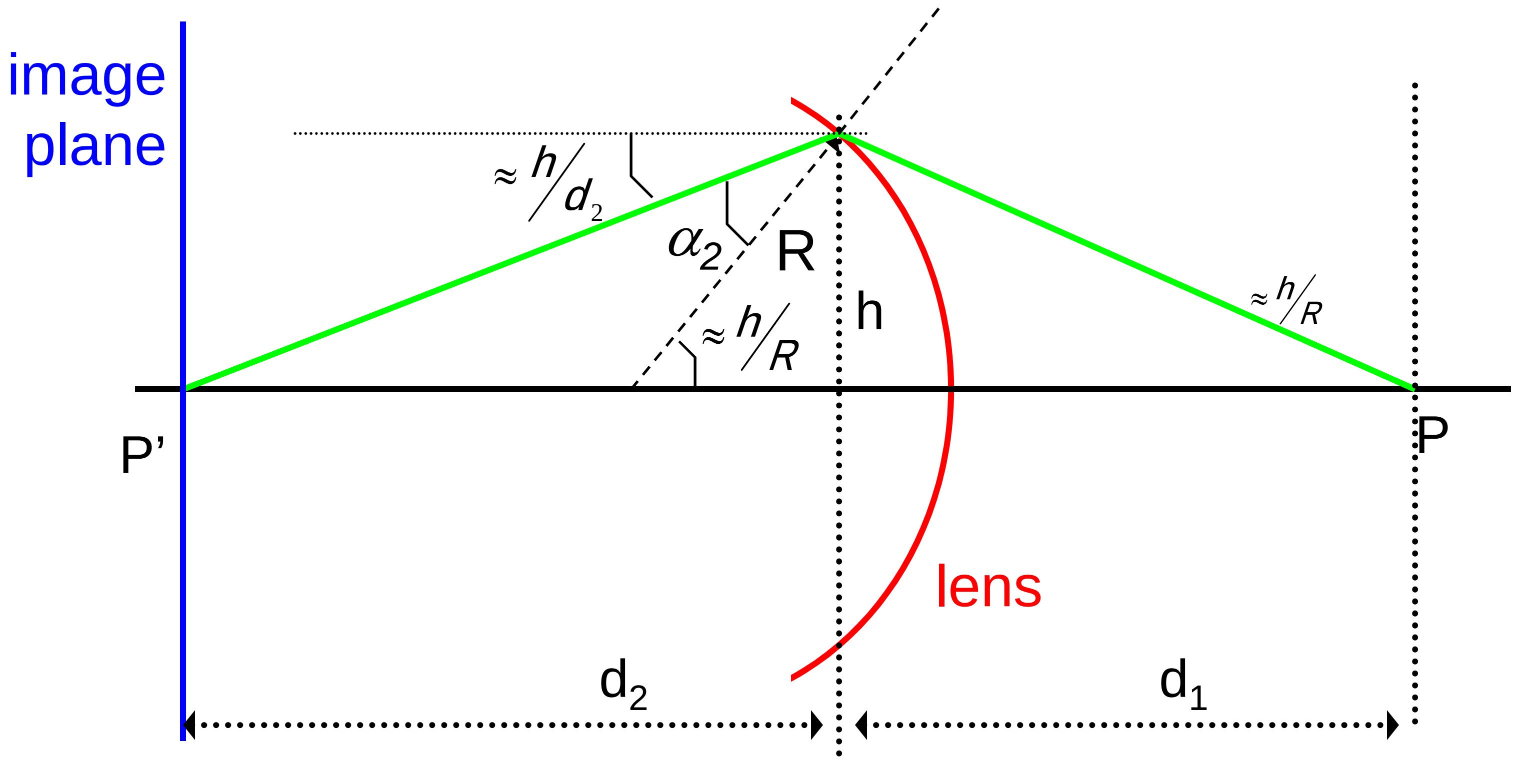
- we assume all angles are small (d_2, h are in microns):
  - $\alpha = \sin\alpha = tg\,\alpha$



$$\alpha_1 \approx \frac{h}{R} + \frac{h}{d_1}$$

Lenses

- we assume all angles are small (d_2, h are in microns):
  - $\alpha = sin\ \alpha = tg\ \alpha$

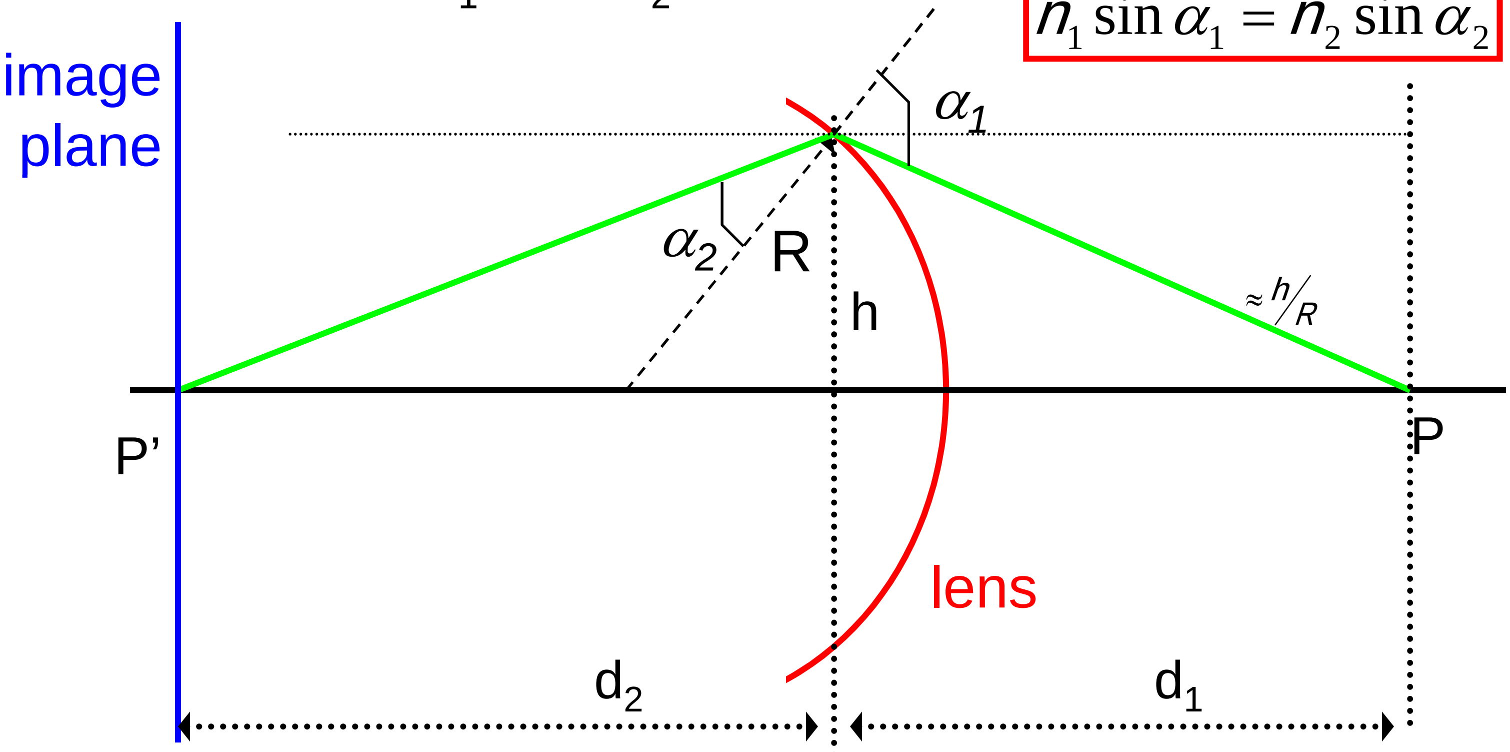


$$\alpha_2 \approx \frac{h}{R} - \frac{h}{d_2}$$

Lenses

- Snell's law
  - for light propagating between two media of indexes of refraction n_1 and n_2

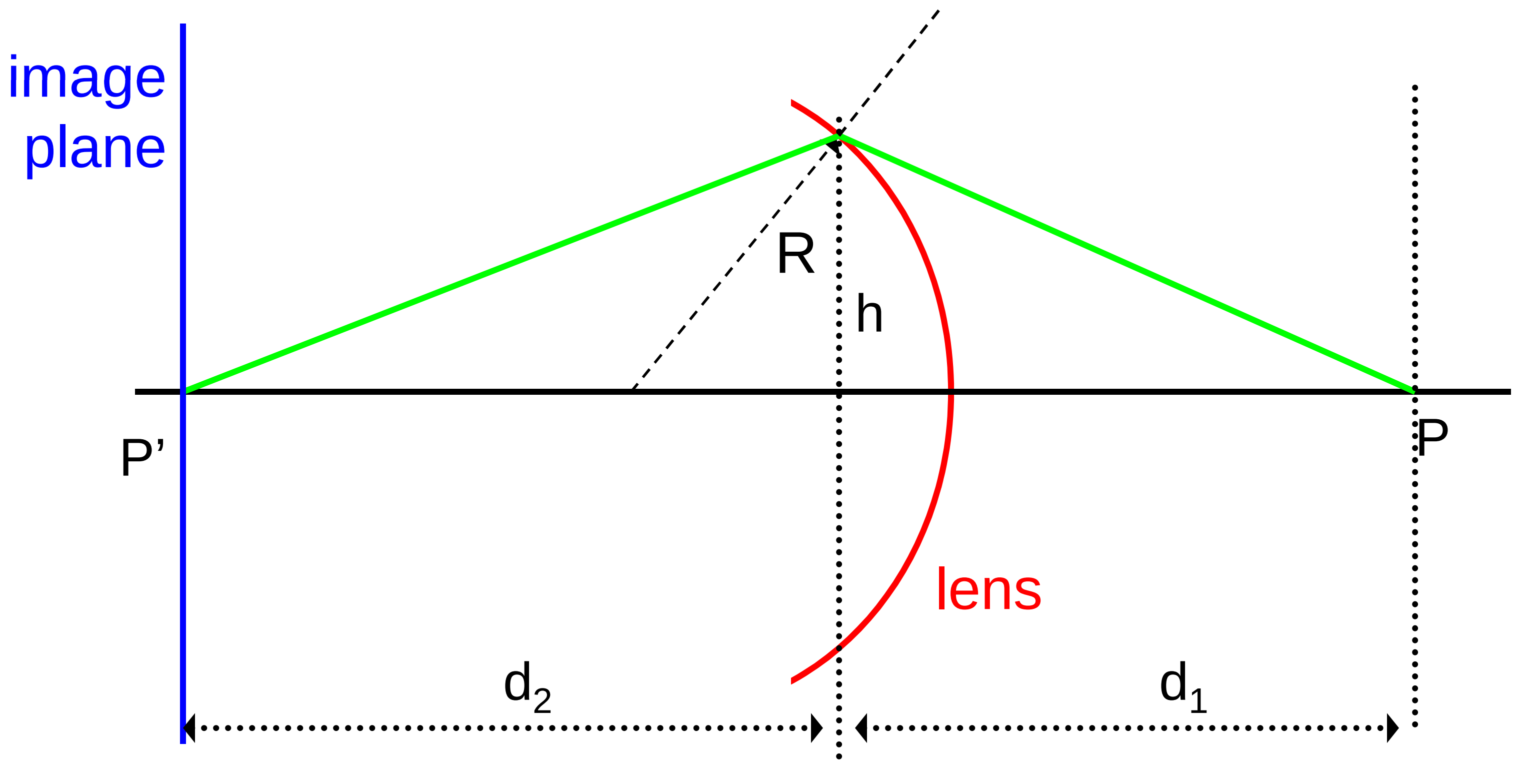
$$n_1 \sin\alpha_1 = n_2 \sin\alpha_2$$



$$\alpha_1 \approx \frac{h}{R} + \frac{h}{d_1} \qquad \alpha_2 \approx \frac{h}{R} - \frac{h}{d_2} \qquad \Rightarrow n_1 h\left(\frac{1}{R} + \frac{1}{d_1}\right) \approx n_2 h\left(\frac{1}{R} - \frac{1}{d_2}\right)$$

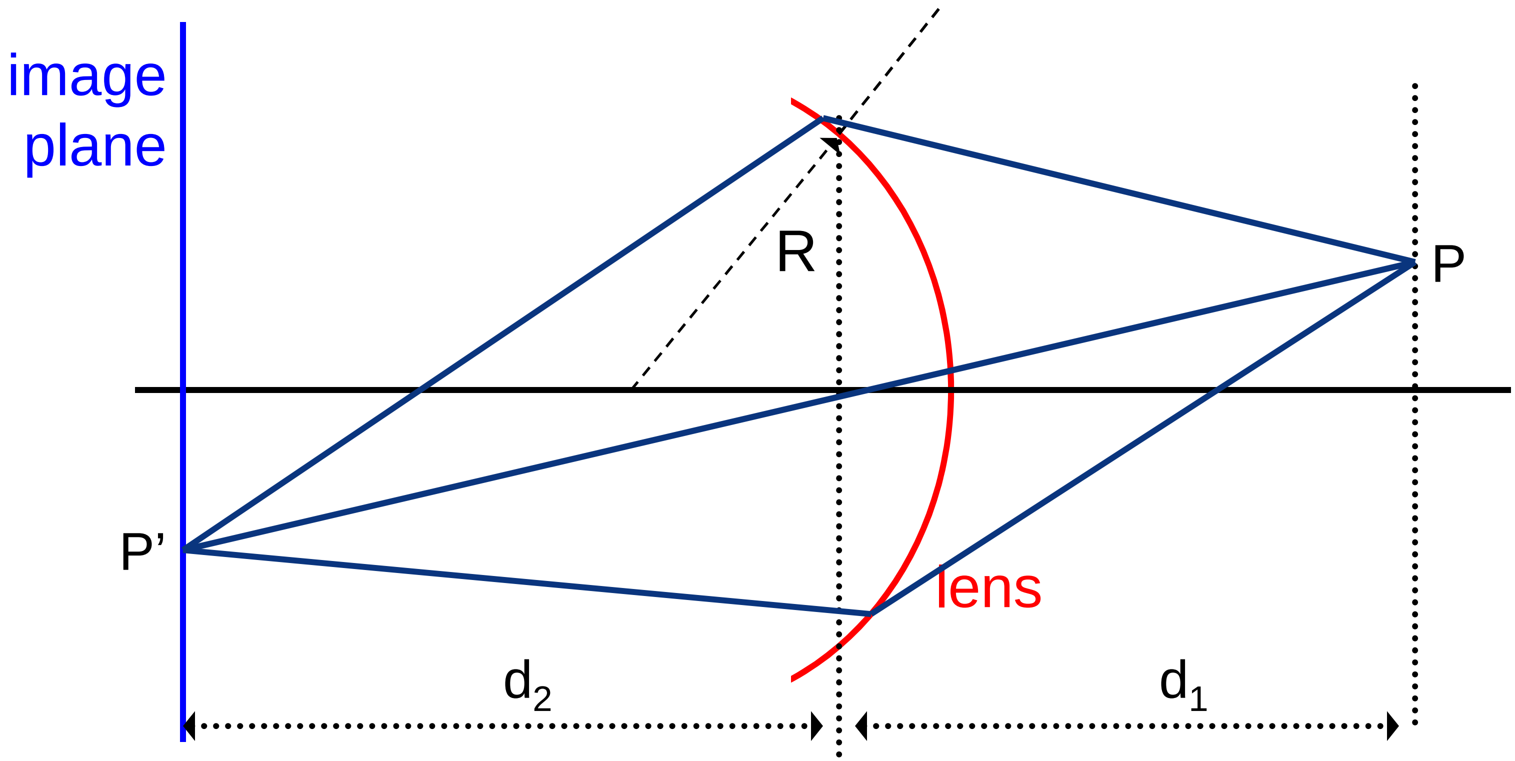
Lenses

- which means that $$\frac{1}{d_1} \approx \frac{1}{n_1}\left(\frac{n_2 - n_1}{R} - \frac{n_2}{d_2}\right)$$
  - given distance d_2, we can compute distance d_1 of the 3D point



Lenses

- which means that $$\frac{1}{d_1} \approx \frac{1}{n_1}\left(\frac{n_2 - n_1}{R} - \frac{n_2}{d_2}\right)$$
  - note that it does not depend on the vertical position of P
  - we can show that it holds for all rays that start in the plane of P



Lenses

- note that, in general,
  - we can only have in focus objects that are in a certain depth range
  - this is why the background is sometimes out of focus on photographs

  - by controlling the focus you are effectively changing the plane of the rays that converge on the image plane without blur

Any questions?