ECE-161C
Color

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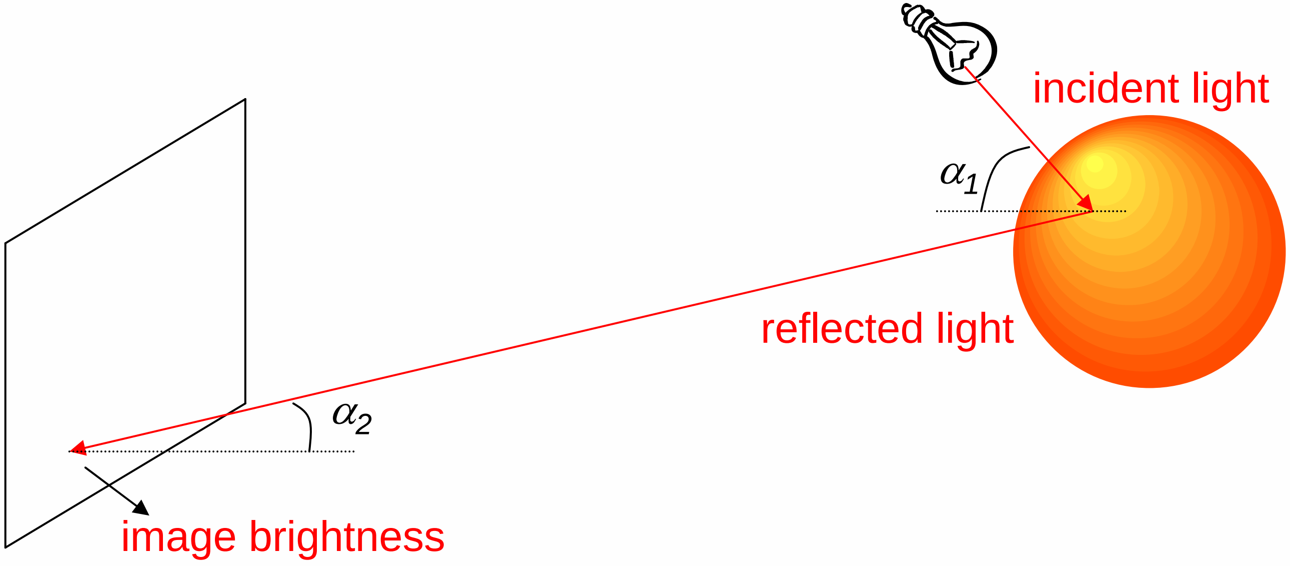
(with thanks to David Forsyth)

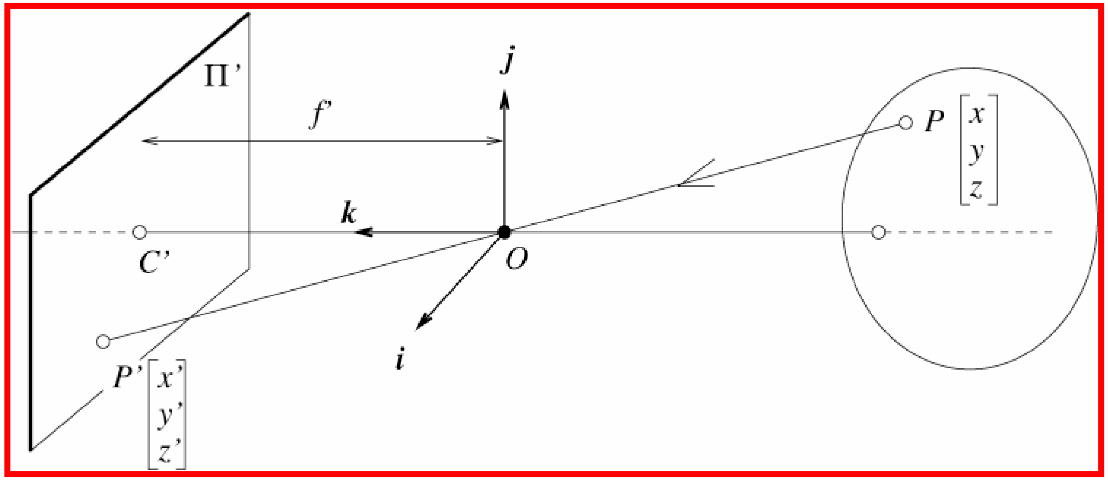
Image formation

- we are studying the process of image formation
- two questions
  - what 3D point projects into pixel (x,y)?
  - what is the light incident on the pixel?
- these determine the image intensity at the pixel



Geometry

- geometry answers the first question
- pinhole camera:
  - point (x,y,z) in 3D scene projected into image pixel of coordinates (x', y')
  - according to the perspective projection equation:

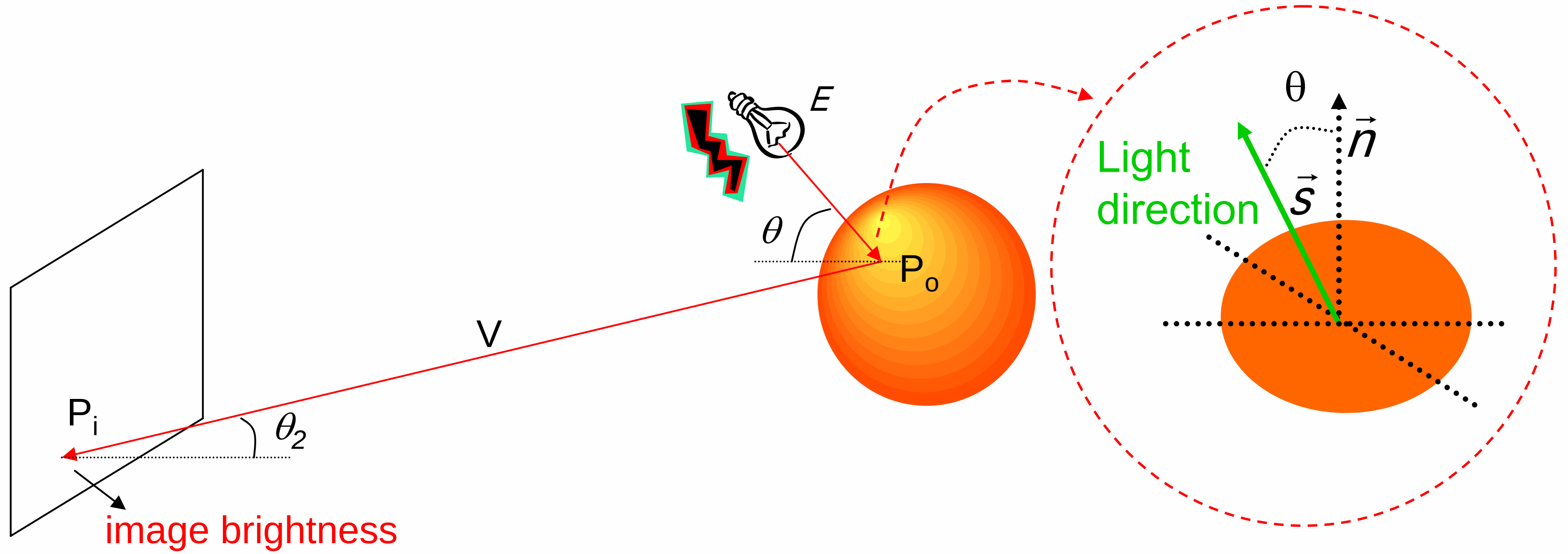


$$\begin{pmatrix} x' \\ y' \end{pmatrix} = f \begin{pmatrix} x/z \\ y/z \end{pmatrix}$$

Lambertian surfaces

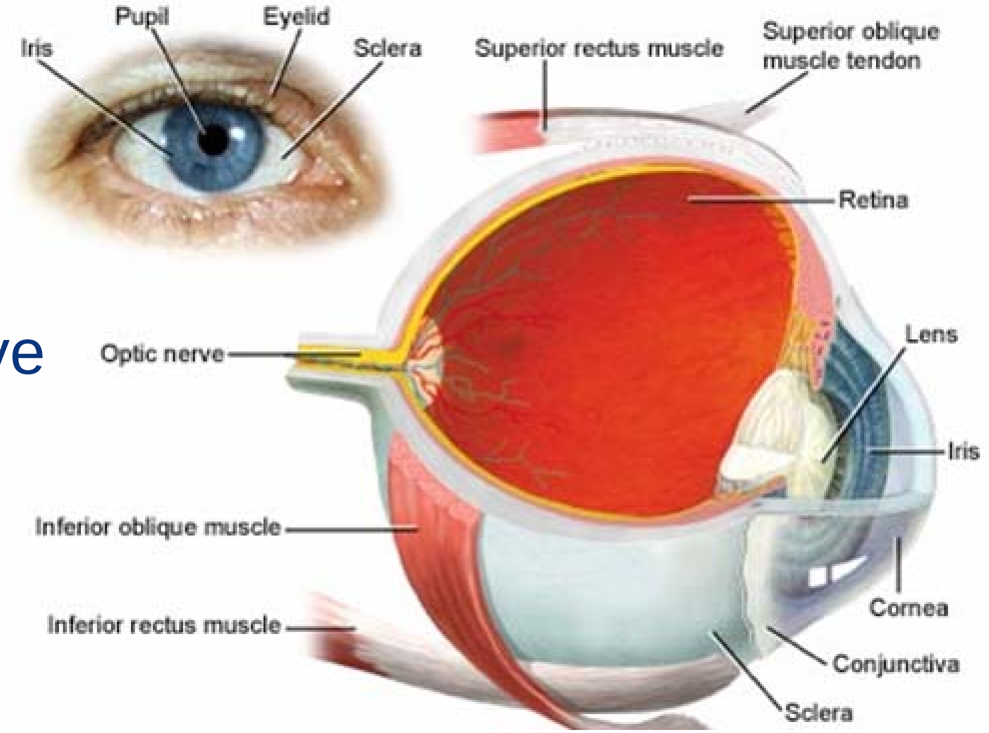
- radiometry answers the second
  - if surface is Lambertian
  - if ***n*** is the surface normal, ***s*** the light direction, ρ the surface albedo, and *E* the source power

$$P(P_i) = E\rho_a(P_0)\vec{n}(P_0).\vec{s}$$



Color

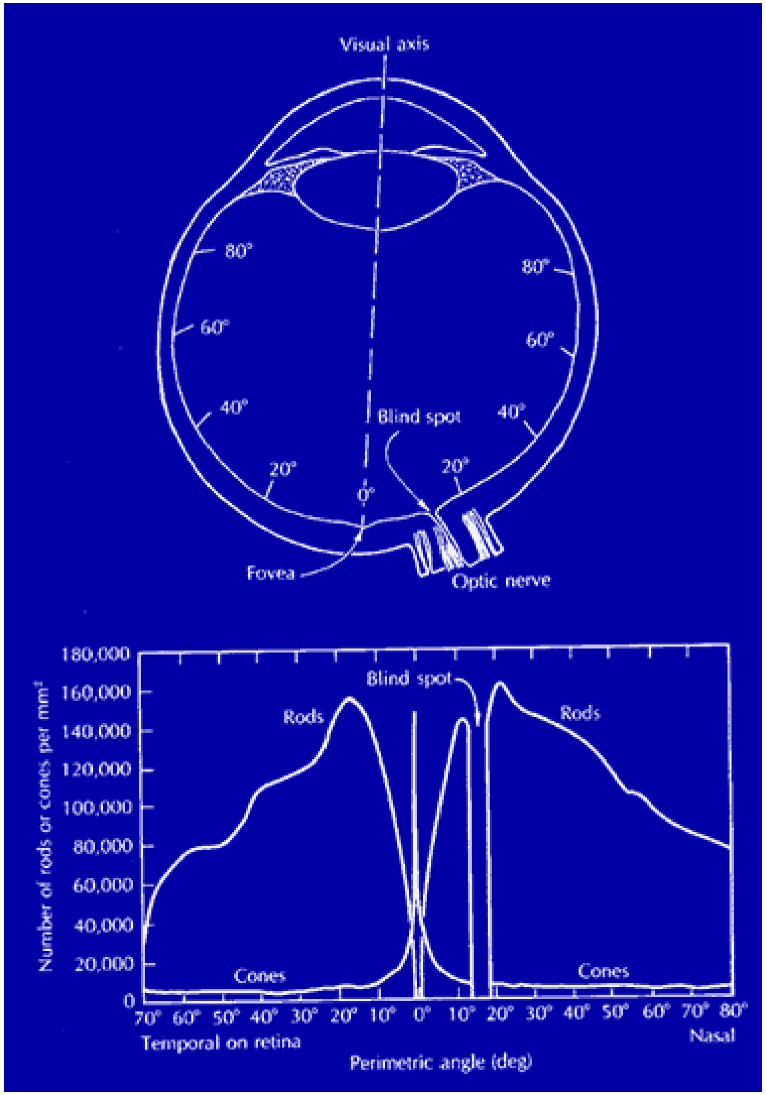
- how do we perceive color?
- color perception is the result of evolution
- let's look at the human eye
  - light enters through pupil
  - this has the role of a lens
  - projects into the retina
  - this is the "camera plane"
  - retina covered by receptors that transform light into electric pulses
  - sent to the brain through the optic nerve



Color

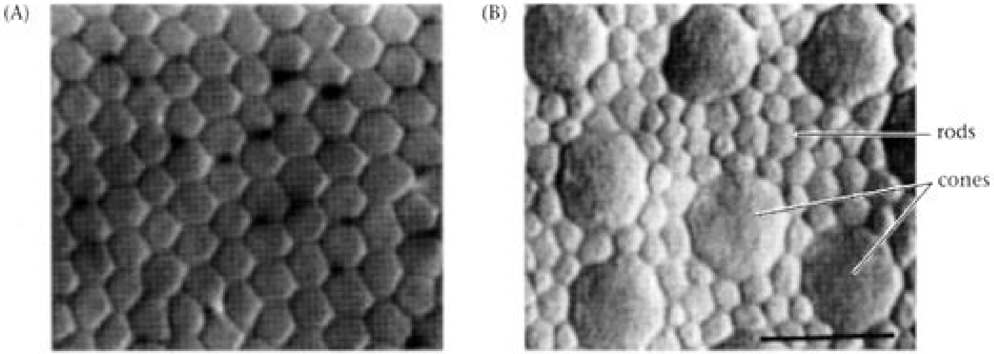
- there are only two types of receptors:
- rods:
  - responsible for vision at low light levels, do not mediate color vision
- cones:
  - active at higher light levels, capable of color vision
- density:
  - rods dominate in the periphery of the eye,
  - the center is mostly composed of cones
- three types of cones:
  - denoted by S, M, and L

Rod vs cone density



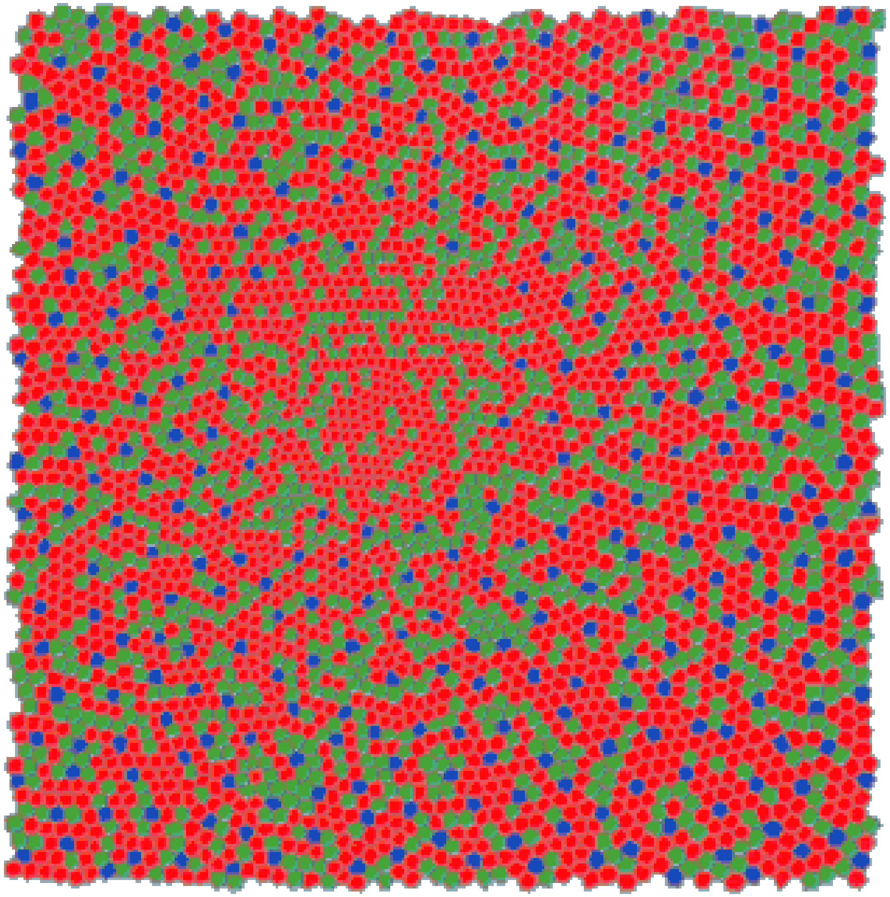
- very high concentration of cones in the center
- high resolution and color
- density decays very quickly

center periphery

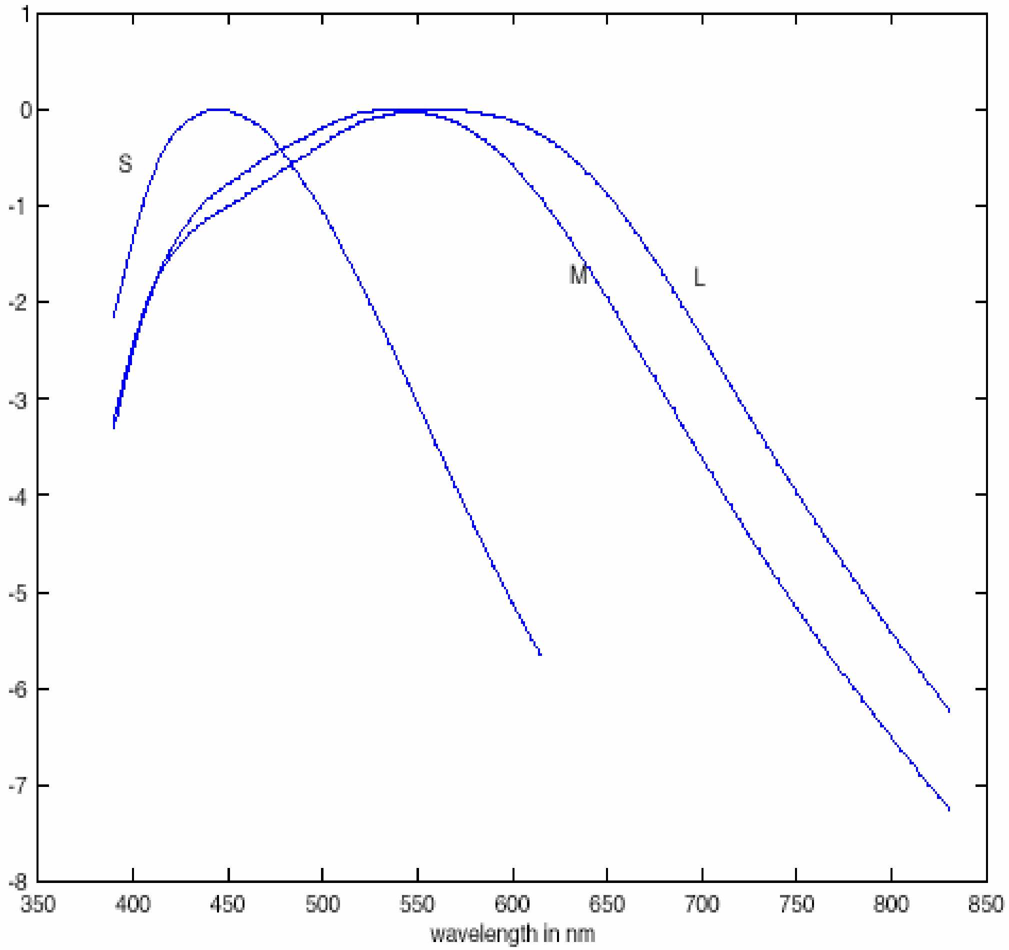


Color receptors

- the three types of cones, vary in their sensitivity to light at different wavelengths
- picture shows a cone mosaic, spatial distribution of the different types of cones
- blue: S cones
  red: L cones
  green: M cones
- note the different densities



Color receptors



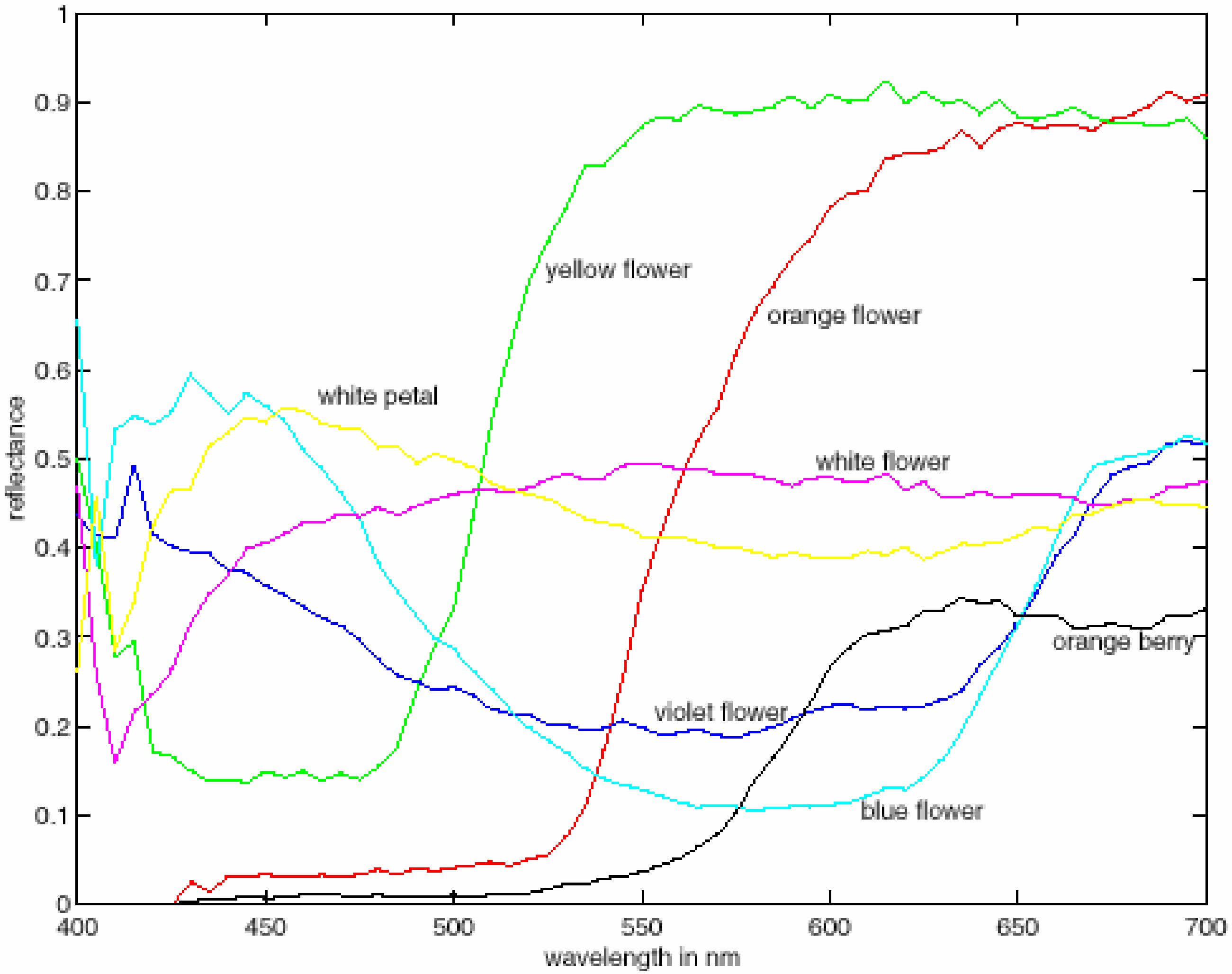
- relative sensitivity as a function of wavelength. S (for short) cone responds most strongly at short wavelengths; the M (for medium) at medium wavelengths and the L (for long) at long wavelengths.
- occasionally called B, G and R cones respectively, but that’s misleading - you don’t see red because your R cone is activated.

Radiometry for color

- the three types of cones are filters, specialized to certain wavelengths
- why does this happen?
- because:
  - the visible spectrum covers a range of wavelengths
  - radiance is a function of wavelength λ
  - e.g. if you put a color filter in front of your lens, only certain wavelengths go through
  - and all objects reflect light differently at different wavelengths
  - this is what makes them look like they have color

$$L(x, V, \lambda)$$

Spectral albedos



important:

- albedo is a function of wavelength

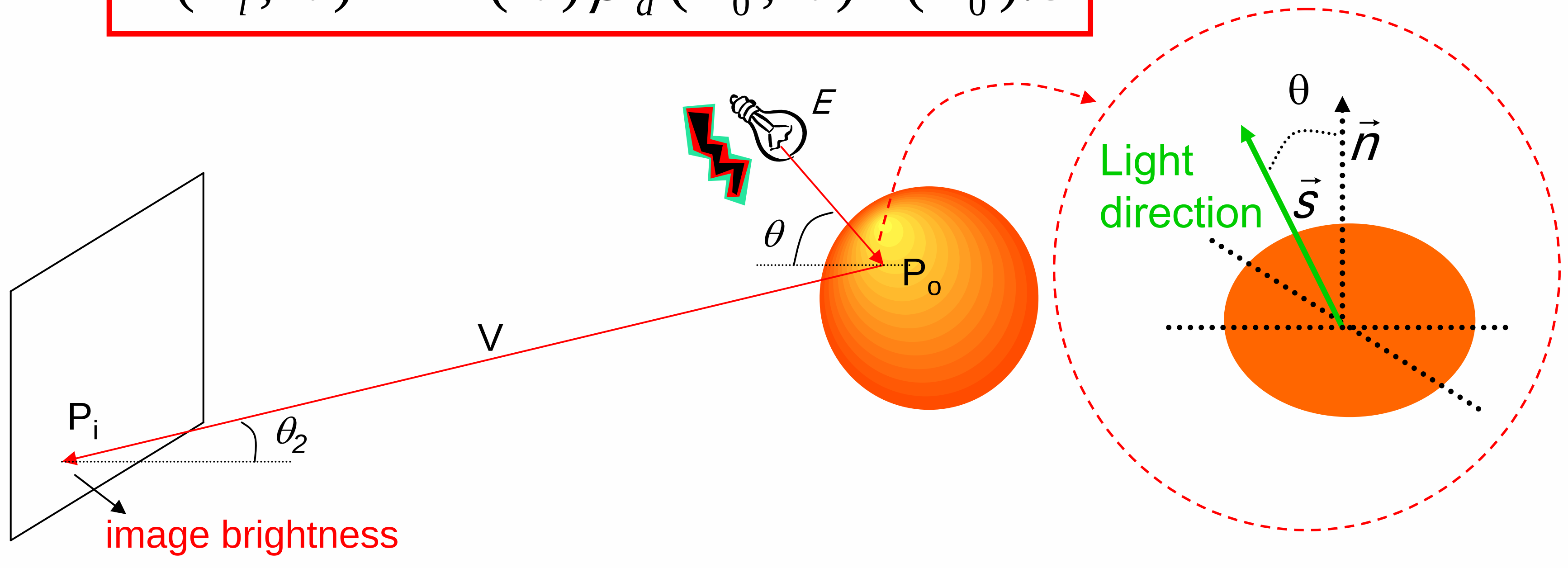
$$\rho(P, \lambda)$$

- example of how it varies with wavelength for different types of leaves (measurements by E.Koivisto).

Spectral albedo

- how does this change our radiometry equation?
  - ***n*** (surface normal), ***s*** (light direction), do not change with wavelength
  - the dependence on wavelength can come from ρ the surface albedo, or *E* the source power

$$P(P_i, \lambda) = E(\lambda)\rho_a(P_0, \lambda)\vec{n}(P_0).\vec{s}$$



Spectral albedo

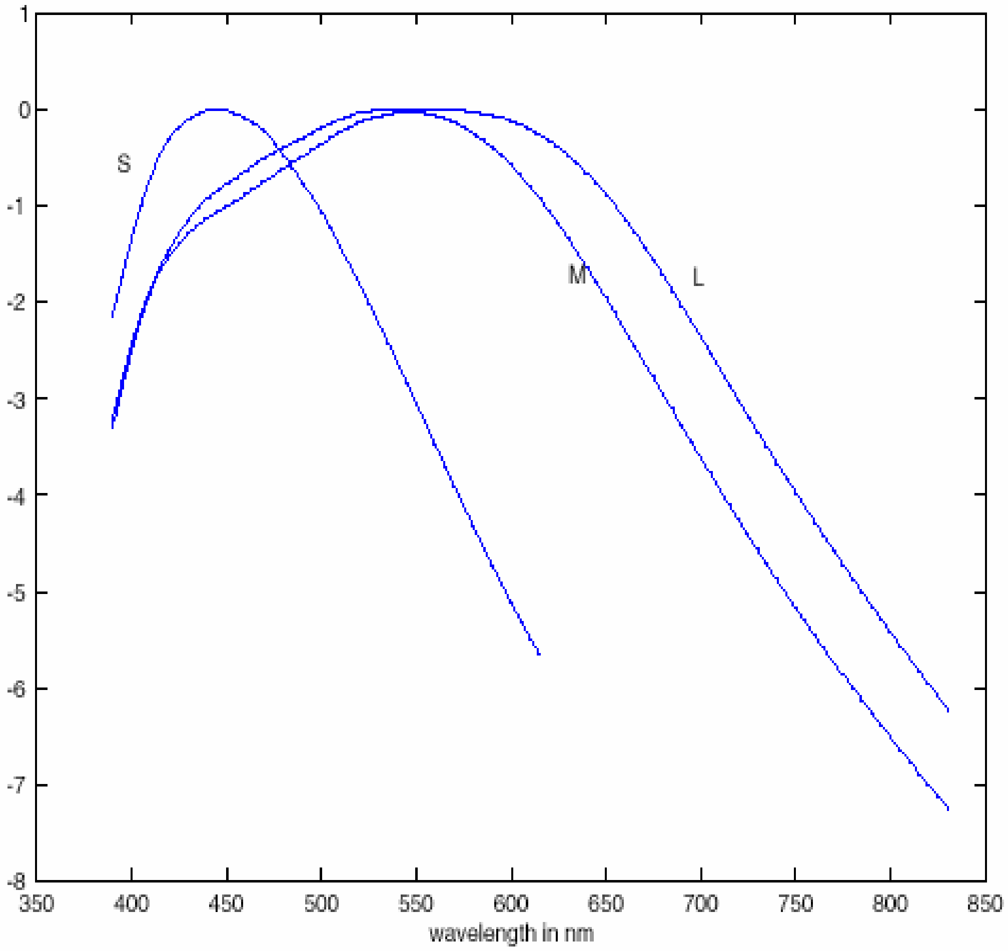
- light
  - can play a significant role in certain environments
  - this is why your perception of color is unreliable in night clubs
- for "white light", *E* is constant
- all dependence is due to the albedo

$$P(P_i, \lambda) = E \rho_a(P_0, \lambda) \vec{n}(P_0).\vec{s}$$

  - e.g. different threads have different albedo
  - color changes from location to location

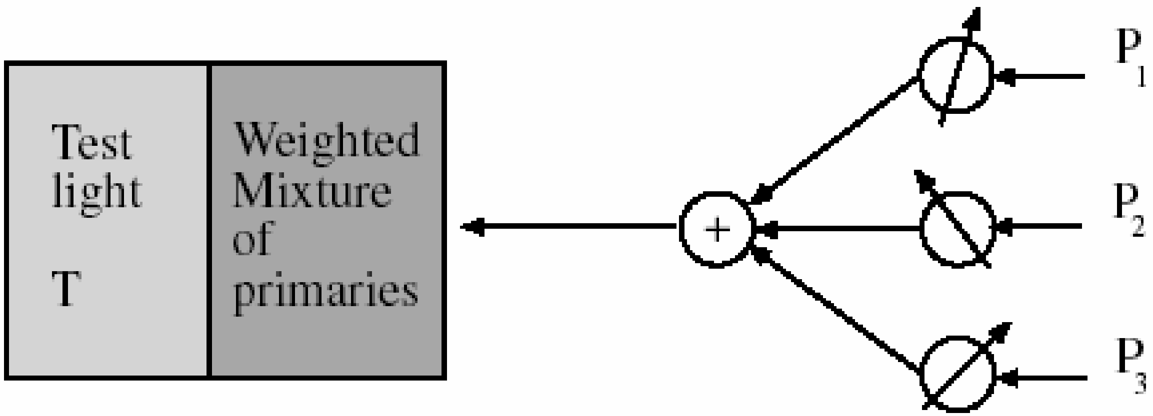
The appearance of colors

- Principle of trichromacy:
  - it is possible to match almost all colors, using only three primary sources.
  - this is not surprising given that we have three types of receptors
  - and these are filters for some wavelengths
  - anything that falls out of the combination of the three primaries can't be seen by us



Color matching experiments

- Subject shown a split field:
  - one side shows the light whose color one wants to measure,
  - other a weighted mixture of primaries (fixed lights).



- Subject adjusts dials so as to make mixture equal to test

Color matching experiments (cont'd)

- most colors can be represented as a mixture of P_1, P_2, P_3

$$M(\lambda) = a\,P_1(\lambda) + b\,P_2(\lambda) + c\,P_3(\lambda)$$

  where the = sign should be read as "matches"

- this is additive matching.
- important because if two people who agree on P_1, P_2, P_3 need only supply (a, b, c) to describe a color.
- some colors can't be matched like this: instead we need

$$M(\lambda) + a\,P_1(\lambda) = b\,P_2(\lambda) + c\,P_3(\lambda)$$

- this is subtractive matching.
- we can interpret it as (-a, b, c)

The principle of trichromacy

- Experimental facts:
- three primaries will work for most people if we allow subtractive matching
  - Exceptional people can match with two or only one primary.
  - This could be caused by a variety of deficiencies.
- most people make the same matches.
  - There are some anomalous trichromats, who use three primaries but make different combinations to match.

Grassman's Laws

- important property: color matching is linear

1. mixture of coordinates matches the mixture of the lights

$$\left.\begin{aligned} T_1 &= a_1 P_1 + b_1 P_2 + c_1 P_3 \\ T_2 &= a_2 P_1 + b_2 P_2 + c_2 P_3 \end{aligned}\right\} T_1 + T_2 = (a_1 + a_2) P_1 + (b_1 + b_2) P_2 + (c_1 + c_2) P_3$$

2. equal coordinates means equal lights

$$\left.\begin{aligned} T_1 &= aP_1 + bP_2 + cP_3 \\ T_2 &= aP_1 + bP_2 + cP_3 \end{aligned}\right\} \; T_1 = T_2$$

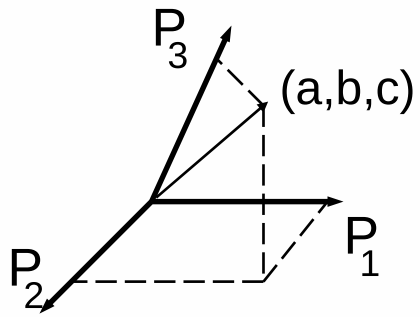
3. matching is linear

$$T_1 = aP_1 + bP_2 + cP_3 \Leftrightarrow kT_1 = kaP_1 + kbP_2 + kcP_{32}$$

- these are known as Grassman's Laws

Linear color spaces

- because color matching is linear in the primaries it makes sense to think of the primaries as the basis of a linear color space
- note that the space is infinite dimensional
  1. think of $L(\lambda)$ as $L(\lambda_1, \lambda_2, \ldots, \lambda_N)$
  2. take N to infinity
- the space of valid colors is a 3D-subspace
- problem: the basis is not necessarily orthogonal



Basis functions

- you are probably used to vector spaces with a finite number of components
  - $f = (f_1, ..., f_n)$
- when n goes to infinity we have a function
  - $f = f(t)$
- this is also a vector space, but now a vector space of functions
  - everything that you have learned still holds, we only need to change our operators a little
  - the main difference is that summations become integrations
  - e.g. for the dot product between f and g

$$< f, g >= \sum_i f_i g_i \quad \Rightarrow \quad < f, g >= \int f(t) g(t) dt$$

Basis functions

- a basis for a subspace of dimension d is a set of d functions $b_1(t), ..., b_d(t)$ such that
  - any function in the space is a linear combination of these d functions (the $b_i(t)$ span the space)

$$f(t) = \sum_{i=1}^{d} \alpha_i b_i(t)$$

  - the $b_i(t)$ are linearly independent (i.e. there is no linear combination that will add to the zero function other than $\alpha_i=0$)
- the basis is orthonormal if the $b_i(t)$ are orthogonal functions and have unit norm

$$\langle b_i(t), b_j(t) \rangle = \int b_i(t) b_j(t) dt = \begin{cases} 1, & if \quad \mathrm{i} = \mathrm{j} \\ 0, & if \quad \mathrm{i} \neq \mathrm{j} \end{cases}$$

Fourier series

- note that you have already seen this
- for example, the Fourier series is the result of the projection of a function f(x) on the basis whose functions are cos(nx) and sin(nx)
- note that the basis is orthogonal, since

$$\int \cos(nx)\sin(mx)dx = 0 \qquad \int \cos(nx)\cos(mx)dx = \begin{cases} \pi, & n = m \\ 0, & n \neq m \end{cases}$$

$$\int \sin(nx)\sin(mx)dx = \begin{cases} \pi, & n = m \\ 0, & n \neq m \end{cases}$$

- the series coefficients are the coordinate on this basis

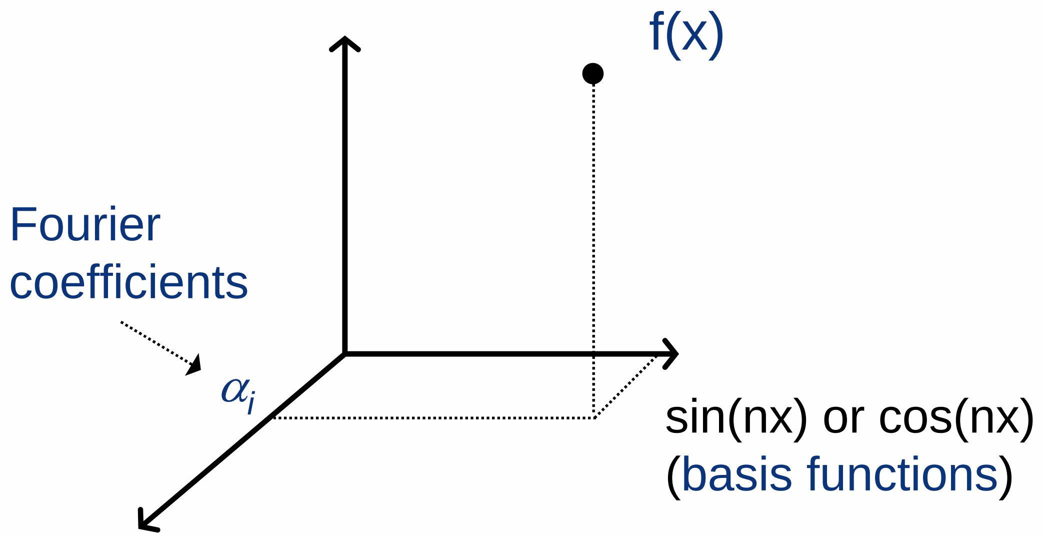
$$\alpha_0 = \frac{1}{\pi}\int f(x)dx \qquad \alpha_n = \frac{1}{\pi}\int f(x)\cos(nx)dx \qquad \beta_n = \frac{1}{\pi}\int f(x)\sin(nx)dx$$

Fourier series

- the function is a linear combination of the basis functions

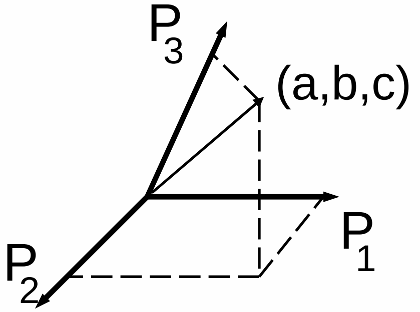
$$f(x) = \frac{1}{2}\alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos(nx) + \sum_{n=0}^{\infty} \beta_n \sin(nx)$$

- note that the basis functions do not have unit norm, and so we need the $1/\pi$ factors
- the picture is



Linear color spaces

- back to color
- because color matching is linear in the primaries it makes sense to think of the primaries as the basis of a linear color space
- note that the space is infinite dimensional
- the space of valid colors is a 3D-subspace
- problem: the basis is not necessarily orthogonal



Color matching functions

- how do we get the color coordinates?
  - pick a source $\delta(\lambda - \lambda_0)$ of unit radiance at wavelength λ
  - we know that there is a set of weights that matches it
  - denote the weight of primary P_i by $f_i(\lambda_0)$

$$\delta(\lambda - \lambda_0) = \sum_{i=1}^{3} f_i(\lambda_0) P_i(\lambda)$$

  - repeat for all λ_0
- the $f_i(\lambda)$ are called color matching functions because we can write

$$L(\lambda) = \int L(\lambda_0)\delta(\lambda_0 - \lambda)d\lambda_0$$

$$= \sum_i \left\{ \int L(\lambda_0) f_i(\lambda_0) d\lambda_0 \right\} P_i(\lambda) = \sum_i \omega_i P_i(\lambda)$$

Color matching functions (cont'd)

- the color coordinates are the projections of the spectral radiance on the three matching functions

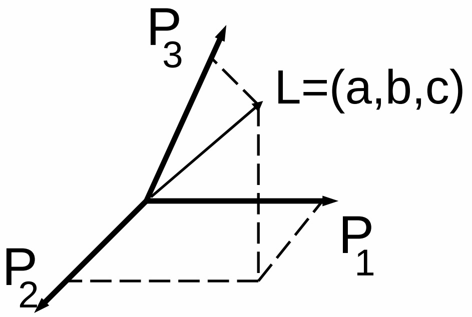
$$a = \int L(\lambda_0) f_1(\lambda_0) d\lambda_0$$
$$b = \int L(\lambda_0) f_2(\lambda_0) d\lambda_0$$
$$c = \int L(\lambda_0) f_3(\lambda_0) d\lambda_0$$

- note that this means that we can specify the color space by specifying a set of matching functions
- this can sometimes lead to primaries that are not physically feasible (e.g. if we constrain the matching functions to be non-negative)
- OK because we really only care about the coordinates

In summary

- color can be represented in different color spaces
  - a color space is defined by a set of three primaries

$$\{P_0(\lambda), P_1(\lambda), P_2(\lambda)\}$$



  - any color is a linear combination of these

$$L(\lambda) = aP_0(\lambda) + bP_1(\lambda) + cP_2(\lambda)$$

  - the coordinates are found by projection onto the matching functions

$$a = \int L(\lambda_0) f_1(\lambda_0) d\lambda_0; \quad b = \int L(\lambda_0) f_2(\lambda_0) d\lambda_0$$

$$c = \int L(\lambda_0) f_3(\lambda_0) d\lambda_0$$

  - the matching functions are the solutions of

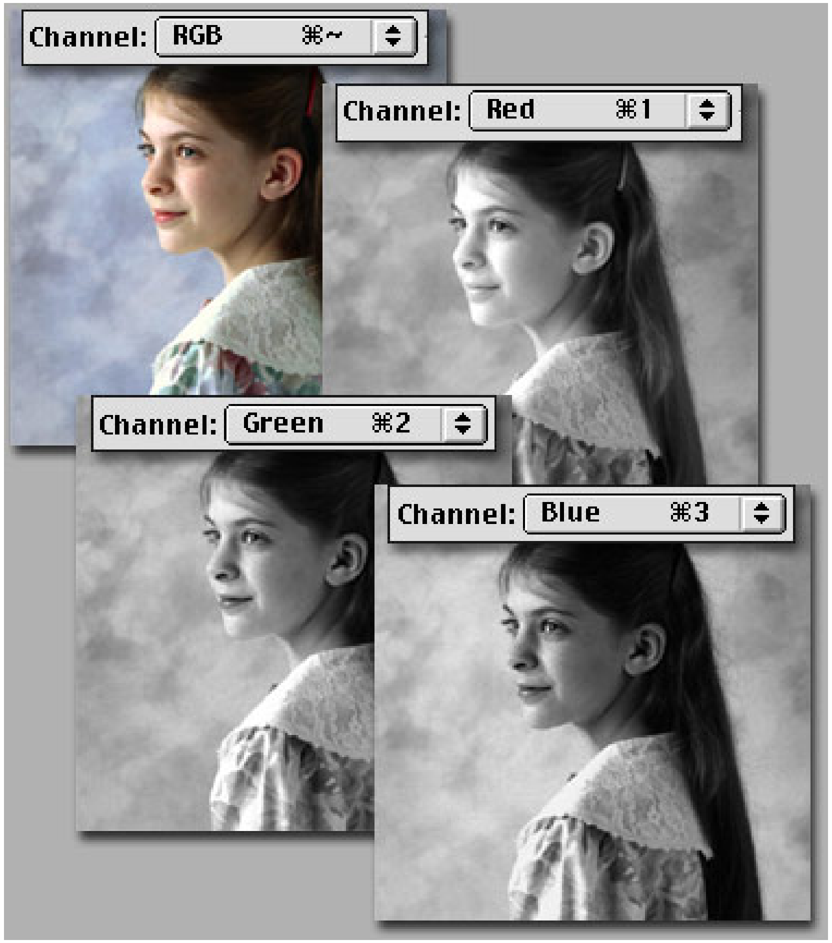
$$\delta(\lambda - \lambda_0) = \sum_{i=1}^{3} f_i(\lambda_0) P_i(\lambda)$$

In summary

- why do we need different color spaces?
  - many reasons
  - some applications require a representation that matches human color perception
  - others need color primaries that are convenient for certain tasks, such as building and LCD or a CRT
- in general
  - we are given the color space, and only work with components (a,b,c)
  - color is a three dimensional vector
  - we have three “images” known as the “three color channels”
  - once in a while, we need to convert between color spaces

in summary

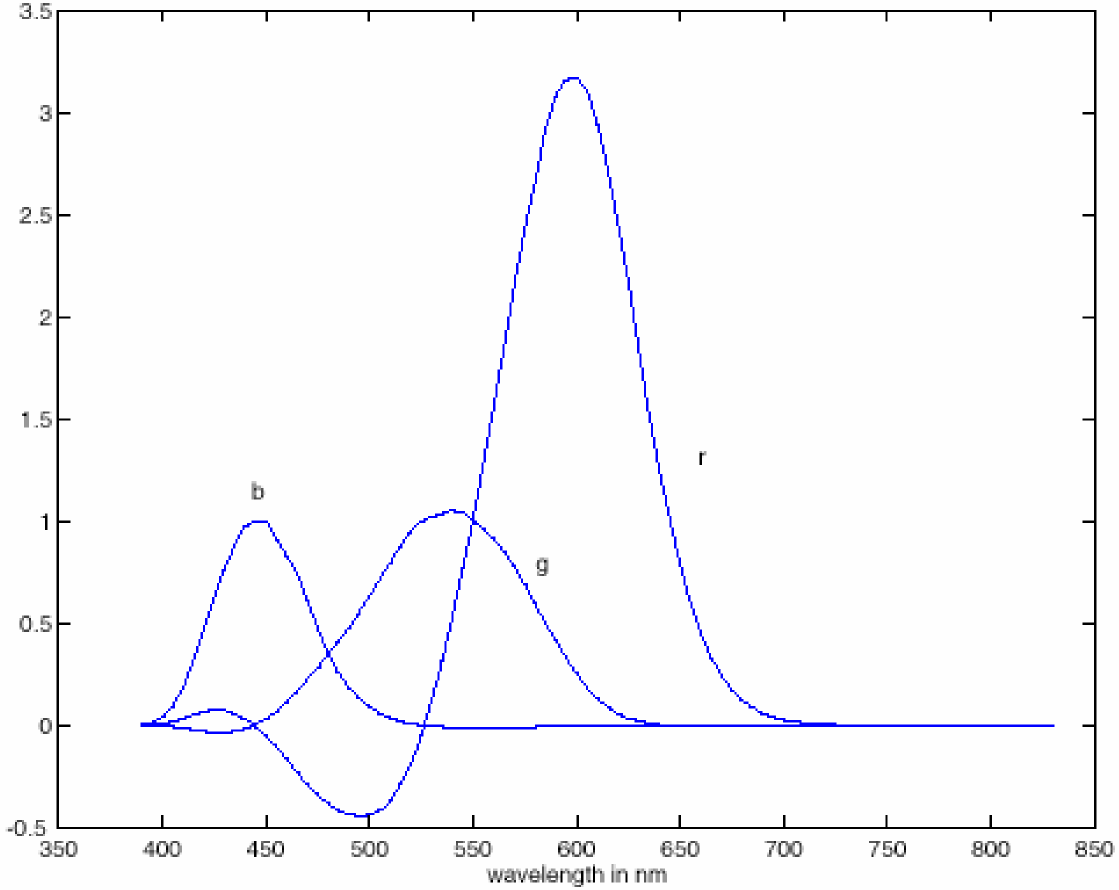
- for example, images are commonly represented in the RGB space
- each pixel is vector of three numbers
- these are the amounts of Red, Green, and Blue
- what exactly is the RGB space?
    - we can look at the associated matching functions



Linear color spaces

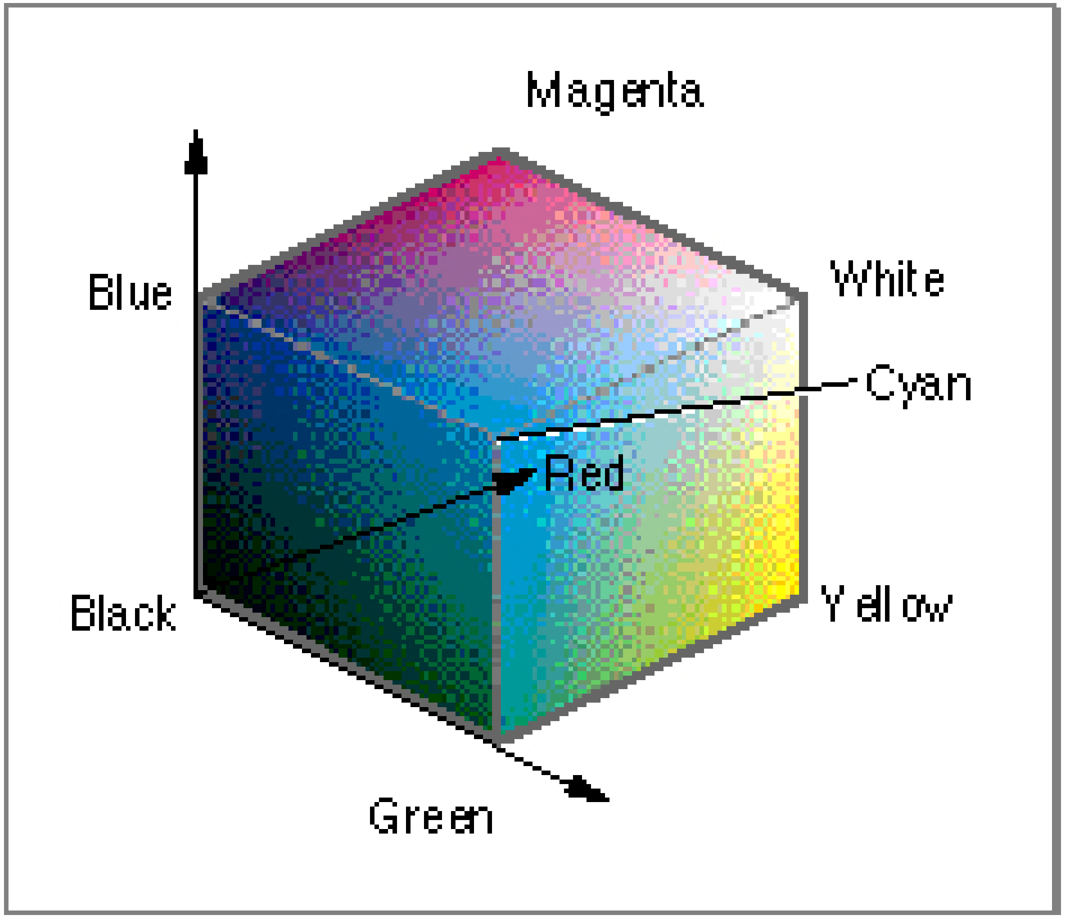
- **RGB:** primaries are monochromatic energies at 645.2nm, 526.3nm, 444.4nm.
- color matching functions have negative parts
- some colors can be matched only subtractively

matching functions



RGB space

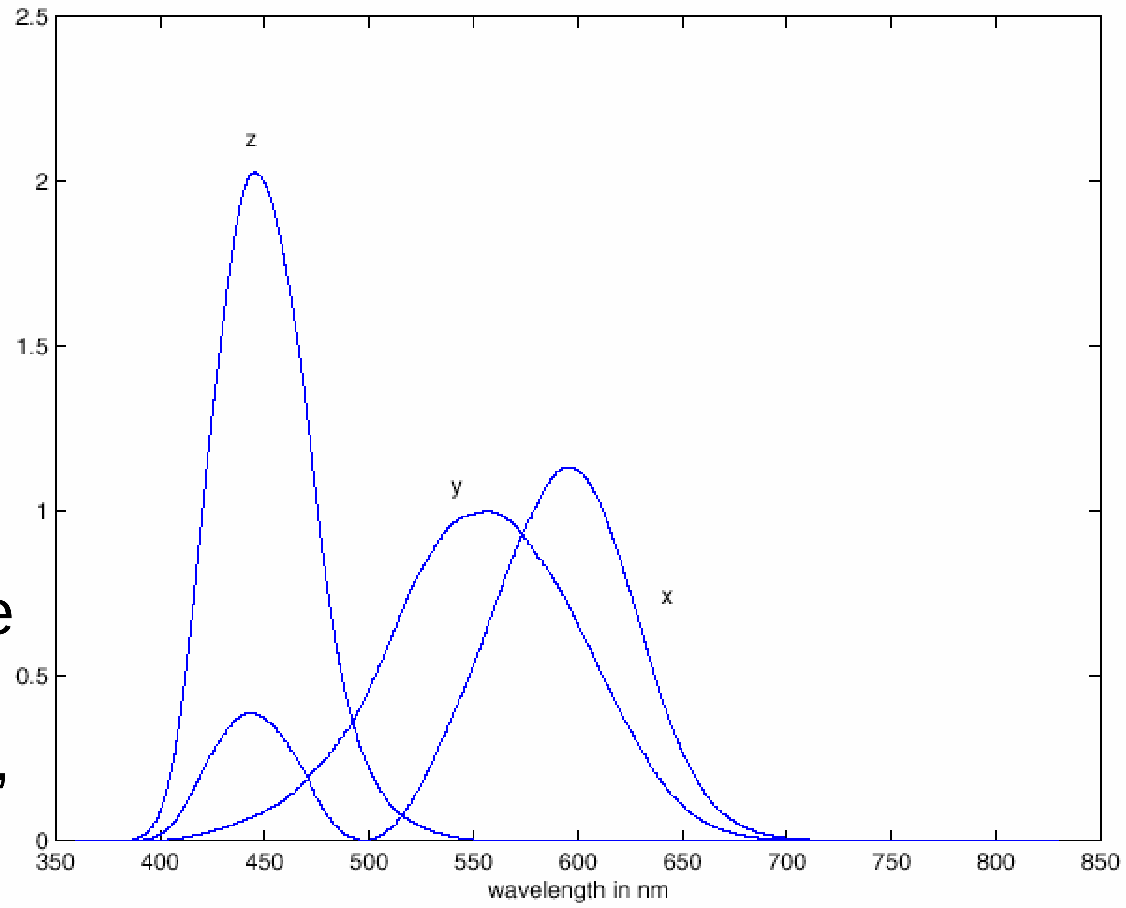
- primaries are the phosphors monitors use as primaries
- the color space is a cube
- axes represent the amount of red, green, and blue
- each color has coordinates between 0 and 1 (0 and 255)



Linear color spaces

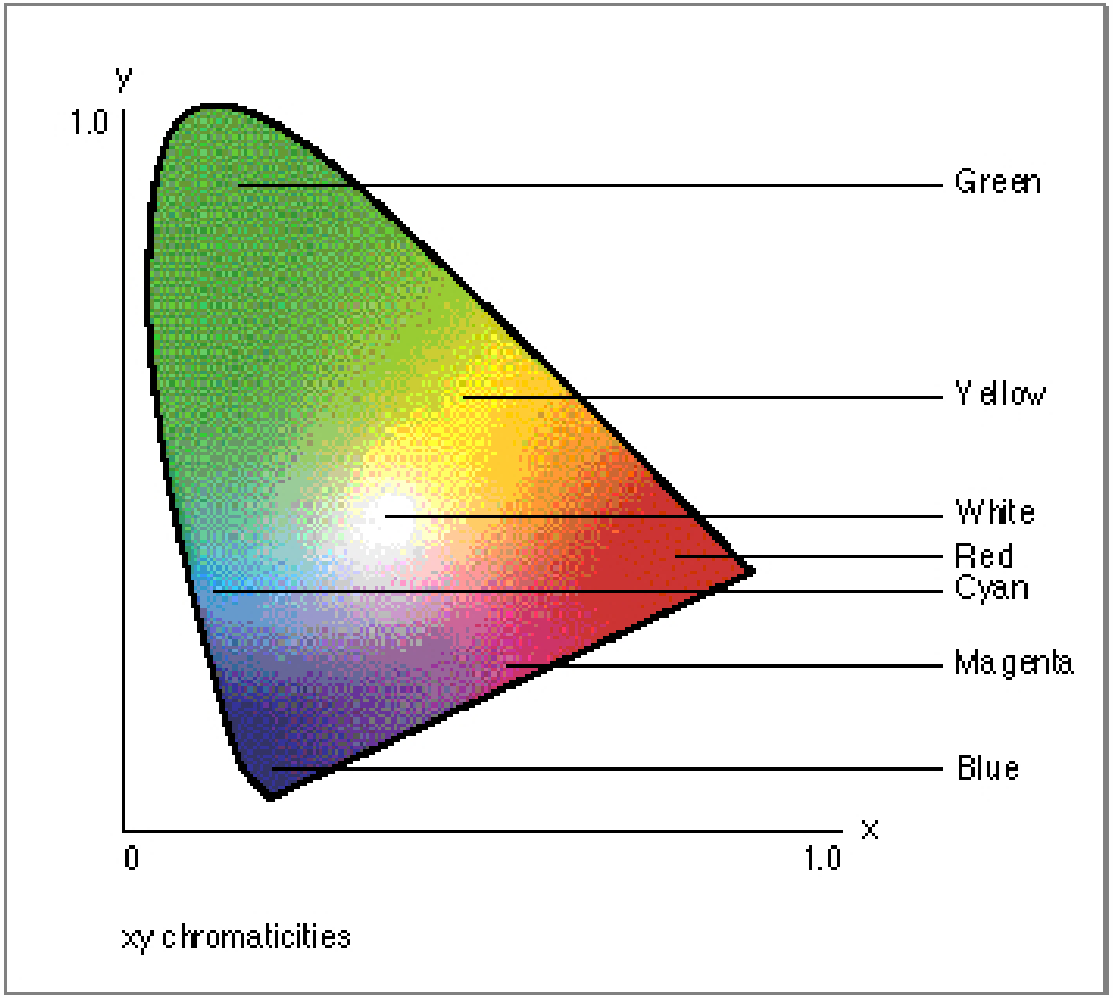
- **CIE XYZ:** color matching functions positive
- due to this primaries are imaginary, but have convenient properties
- color coordinates are (X,Y,Z), where X is amount of X primary, etc.

matching functions



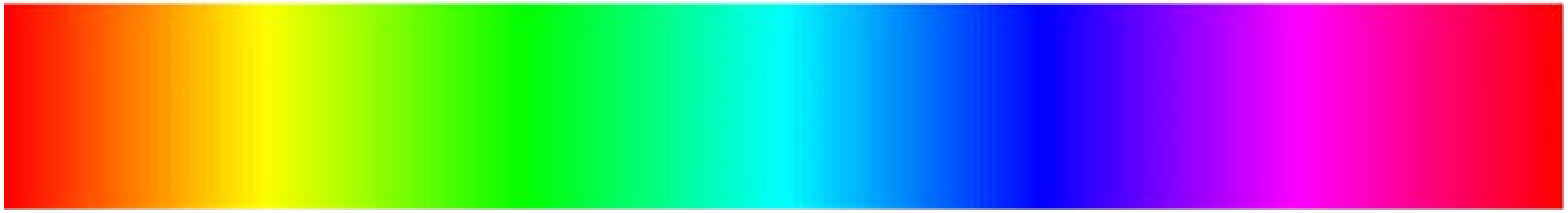
CIE xy

- 2D is easier to visualize than 3D
- usually work with CIE xy, where
  - $x=X/(X+Y+Z)$
  - $y=Y/(X+Y+Z)$
- this is the intersection of the color space with the plane $X+Y+Z=1$



Perceptual color spaces

- human perception of color is best described in terms of three fundamental color properties
- **hue**: this is what you would refer to as the color itself

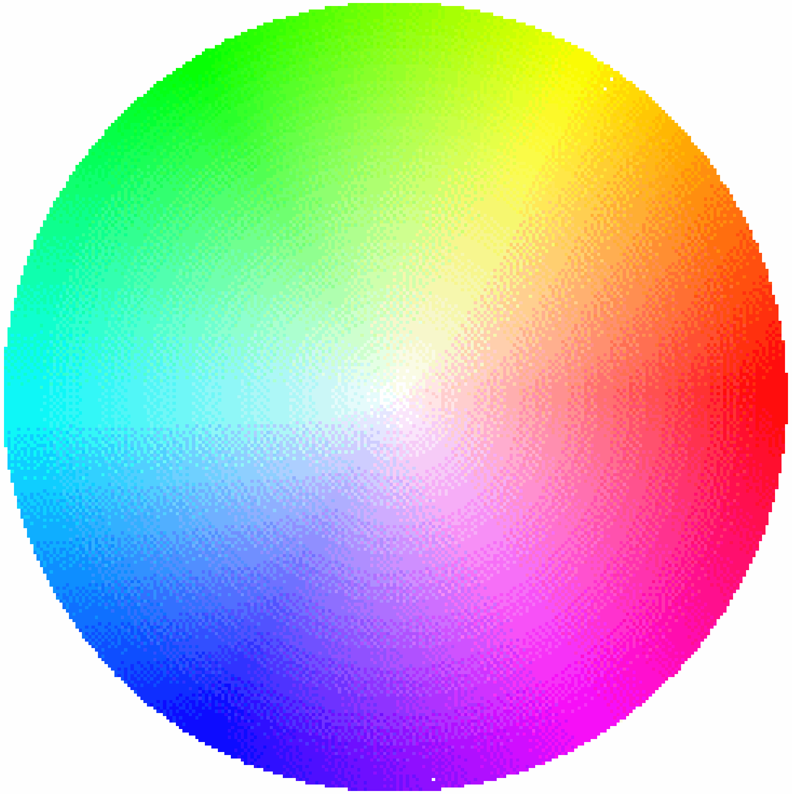


  e.g. red vs yellow

- note that there is a circular structure (we start and finish with red)
- for this reason color is usually visualized in a color wheel

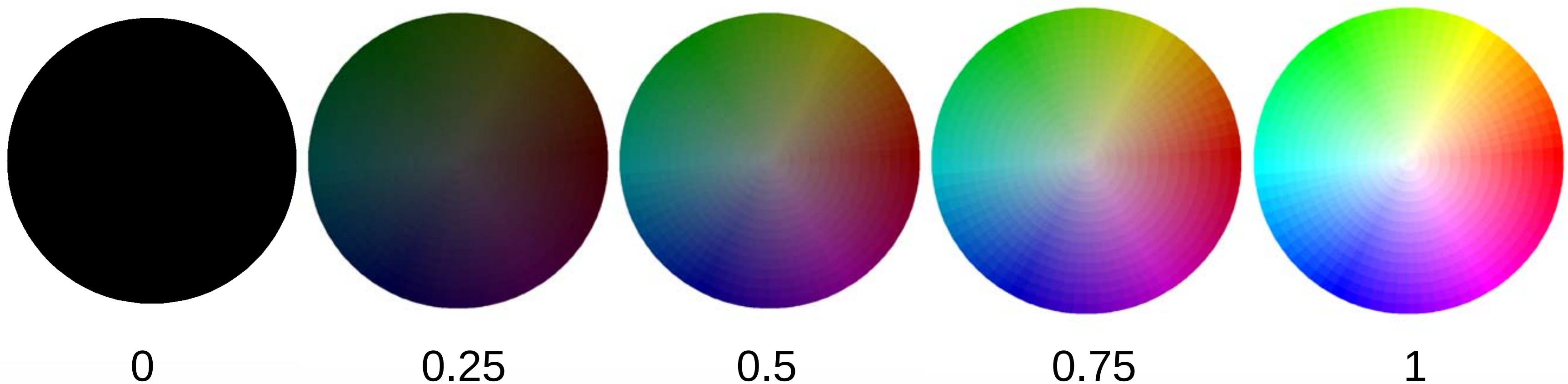
Color perception

- **saturation**: this is the adjective: e.g. “vivid” red, “pale” yellow
- it is the radial coordinate on the color wheel
- colors at the center are unsaturated, colors at the boundaries are highly saturated



Color perception

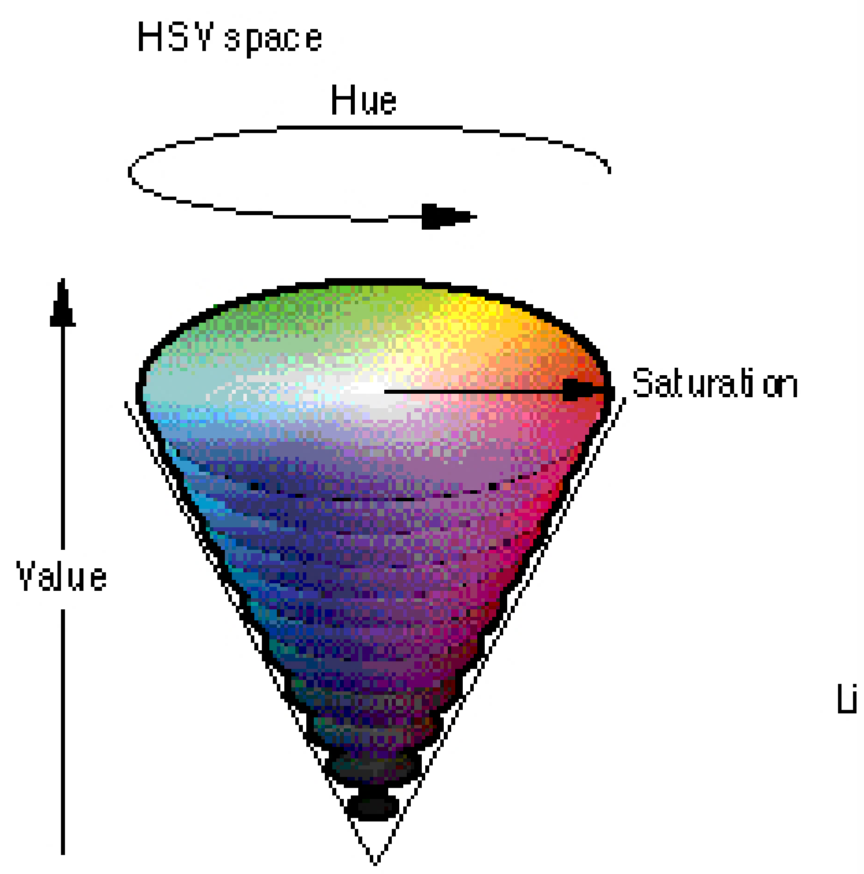
- **intensity:** is the amount of brightness
- “dark” yellow vs “light” yellow
- the color wheel can be replicated at each intensity level



- Note that:
  - the center is always the intensity level
  - at zero intensity, hue and saturation are irrelevant
  - saturation becomes more important at higher intensities

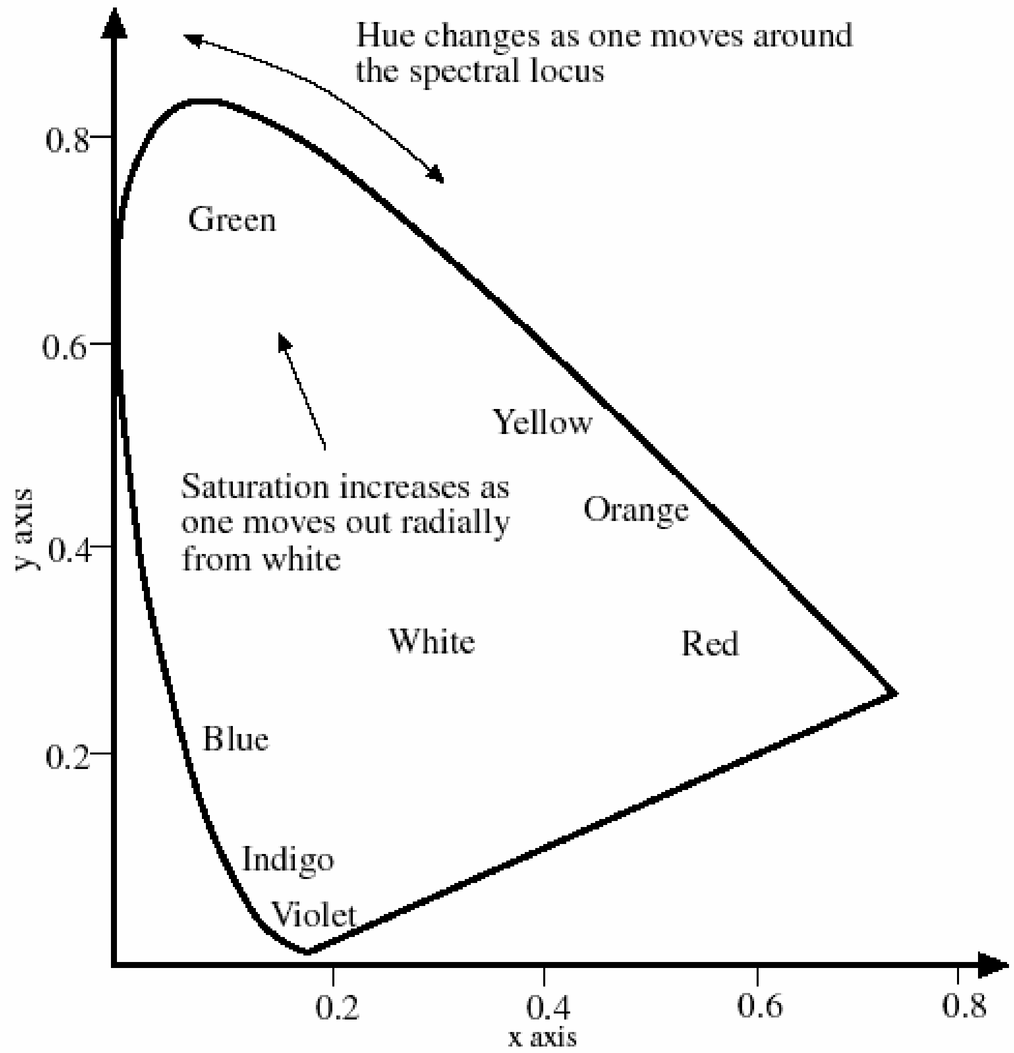
Perceptual color spaces

- **HSV:** hue, saturation, value (intensity) is a natural "perceptual" color space
- very non-linear, colors form a cone
- this can be bad for some applications
- e.g. TVs where primaries have nothing to do with perception but with building CRTs
- but very good for others

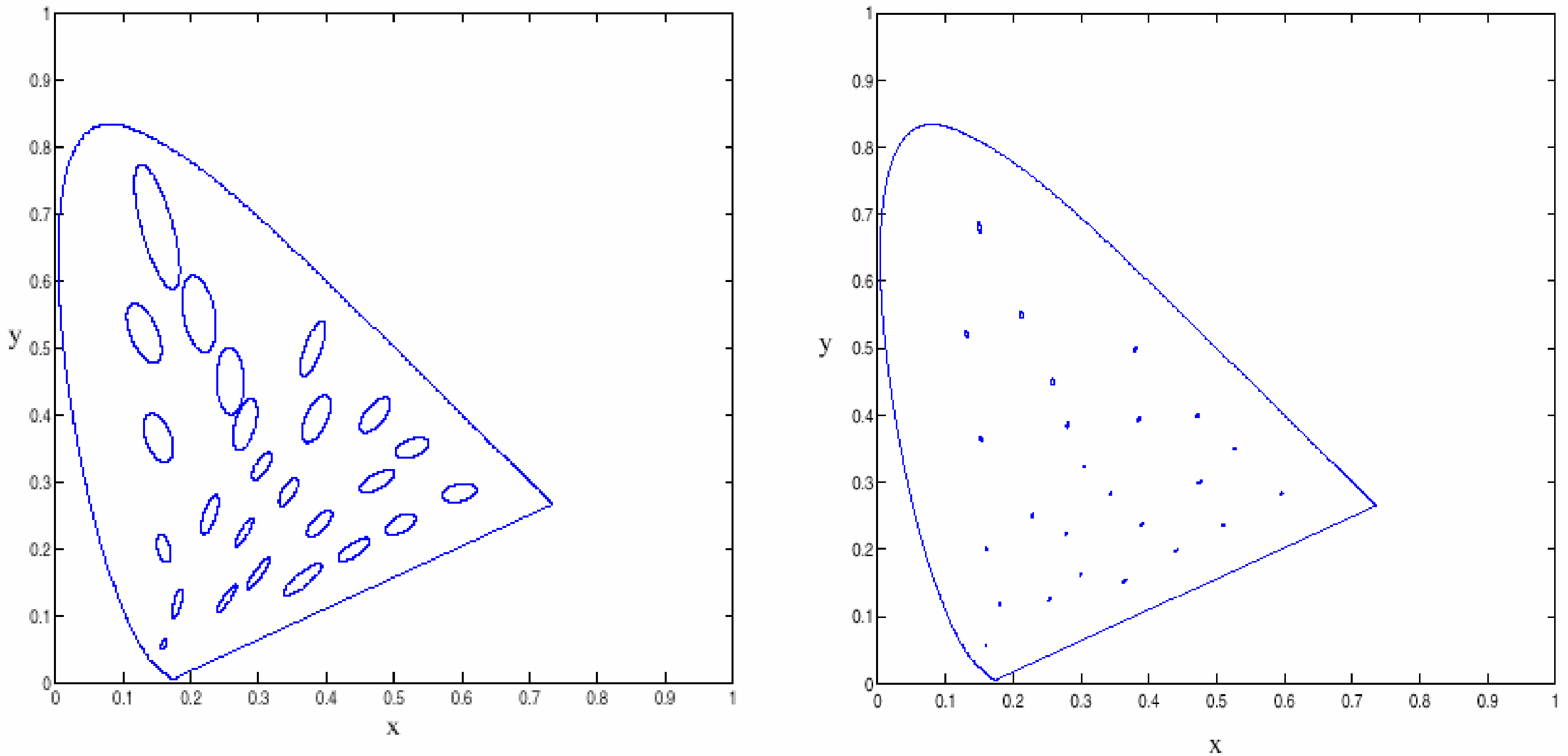


Perceptual color spaces

- note that, in general, we can extract the three properties from any space
- this picture shows H,S, and V in CIE xy
- the problem is that the transformation is non-linear
- in CIExy two colors that are near do not necessarily “look” similar



MacAdam elipses



- ellipses: colors that human observers matched to color on their center; boundary shows just noticeable difference.
- left: magnified 10x. The ellipses at the top are larger than those at the bottom, and rotate as they move up.
- difference in x,y is poor guide to color difference!

Perceptual color spaces

- Uniform: equal (small!) steps give the same perceived color changes
- this is important, for example for image retrieval based on color similarity

- compute an histogram of color values for each image and measure image similarity by the similarity of the color histograms
- works surprisingly well (more on this later)

Conversions

- in practice, we often have to do color-space conversions
- here are some of the formulas (for more details see http://www.easyrgb.com/math.html)
  - RGB to XYZ: see homework
  - CIE XYZ to CIE u'v'

$$(u',v') = \left(\frac{4X}{X + 15Y + 3Z}, \frac{9Y}{X + 15Y + 3Z} \right)$$

  - CIE XYZ to L*a*b

$$L^* = 116\left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - 16 \quad a^* = 500\left[\left(\frac{X}{X_n}\right)^{\frac{1}{3}} - \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}}\right] \quad b^* = 200\left[\left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - \left(\frac{Z}{Z_n}\right)^{\frac{1}{3}}\right]$$

(X_n, Y_n, Z_n) are (X,Y,Z) coordinates of a reference white patch.

Conversions

- RGB to CMY

$C = 1-R \qquad M = 1-G \qquad Y=1-B$

- CMY to CMYK

$K = \min(C,M,Y) \quad C = C-K \qquad M = M-K \qquad Y=Y-K$

- RGB to YCrCb

$Y = 0.29900R + 0.58700G + 0.11400B$

$C_b = -0.\ 16874R - 0.33126G + 0.50000B$

$C_r = 0.50000R-0.41869G - 0.08131B$

Conversions

- RGB to HSV

Min = min(R,G,B); Max = max(R,G,B);

Δ = Max – Min; ***V*** *= Max;*

```
if ( Δ == 0 ) {H = 0; S = 0}
else {
   S = Δ / Max;

   ΔR = [(Max - R) / 6  + (Δ / 2) ] / Δ;
   ΔG = [(Max - G) / 6  + (Δ / 2) ] / Δ
   ΔB = [(Max - B) / 6  + (Δ / 2) ] / Δ

   if ( R == Max ) H = ΔB - ΔG
   else if ( G == Max ) H = ( 1 / 3 ) + ΔR - ΔB
   else if ( B == Max ) H = ( 2 / 3 ) + ΔG – ΔR
   if ( H < 0 ) ; H += 1; if ( H > 1 ) ; H -= 1
}
```

Any questions?