Filtering, scale, orientation, localization, and texture

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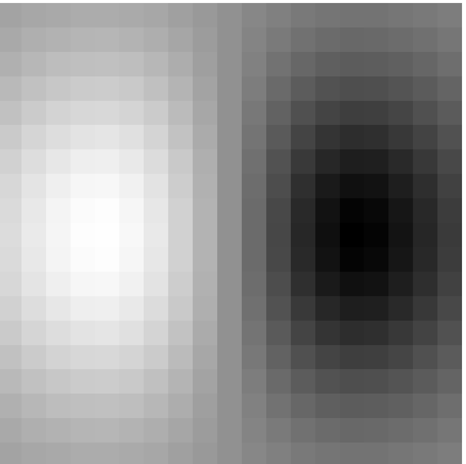
(with thanks to David Forsyth)

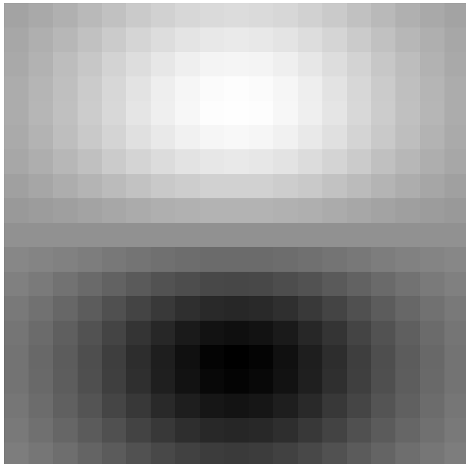
Beyond edges

- we have talked a lot about edges
- while they are important, it is now recognized that they are not enough
- for many objects they are not even defined (think of a landscape with green grass, a mountain, and sky)
- fortunately there is a lot more we can do with filtering than look for edges
- today we will talk about such extensions
- let's start by recalling that filters are templates

Filters as templates

- applying a filter at some point can be seen as taking a dot-product between the image and some vector
- filtering the image is a set of dot products
- insight
  - filters look like the effects they are intended to find
  - filters find effects they look like
- Q: what are good filters (detectors) for vision?





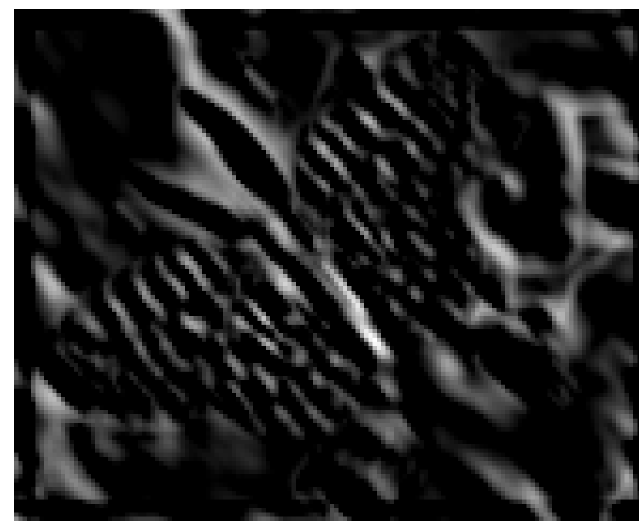
Filters for vision

- the answer is that it depends on what you are looking for
- in the absence of good answers we can look for them in biological vision
- it has been observed that cells in early stages of the brain tend be respond to either
  - spots (e.g. a white spot surrounded by black)
  - bars (e.g. half the cell exposed to white, the other to black)
- this has drawn a lot of attention to spots and bars, which book emphasizes, but they do not have to be the ones
- e.g. if you are working with man-made scenes, e.g. buildings, it is probably a must to detect corners
- but they do illustrate the concept of filters as detectors

spot filter

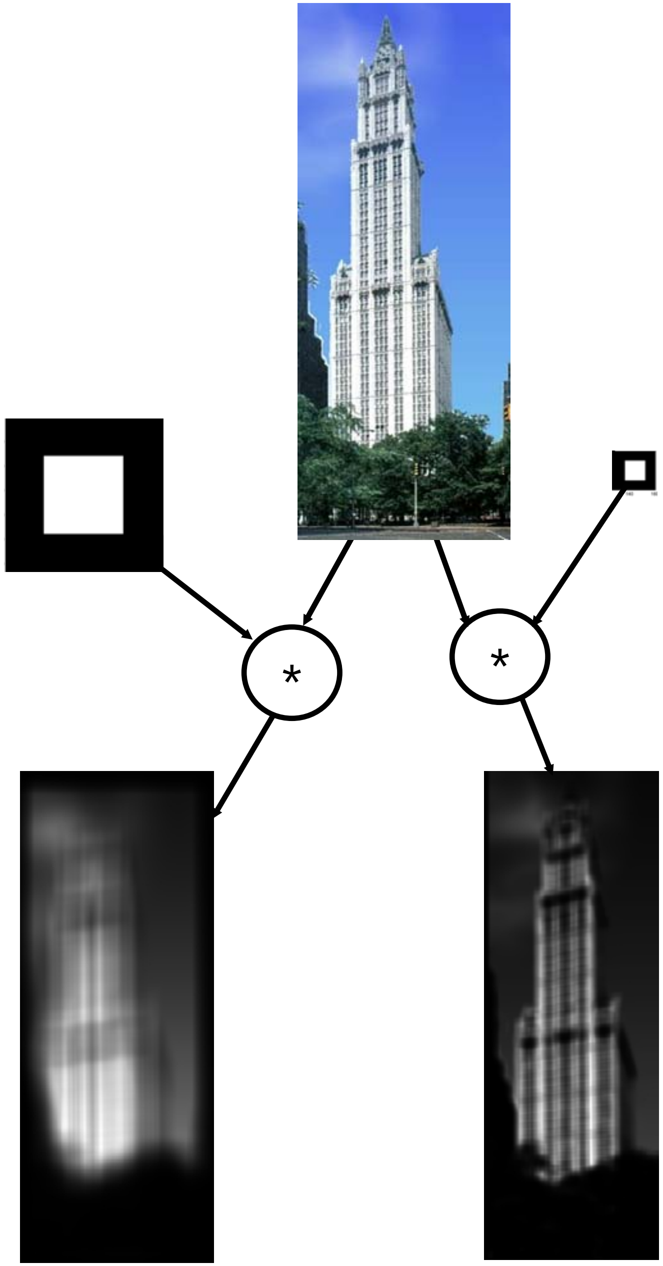
Positive responses

bar filter



Scale and orientation

- non-controversial: apply a set of filters that cover a broad range of scales and orientations
- a large filter will detect coarse object properties, e.g. a building is a box
- a small filter will detect details, e.g. this box contains many little boxes (windows)
- the same with orientation: buildings are vertical boxes, cars are horizontal
- this is a general principle

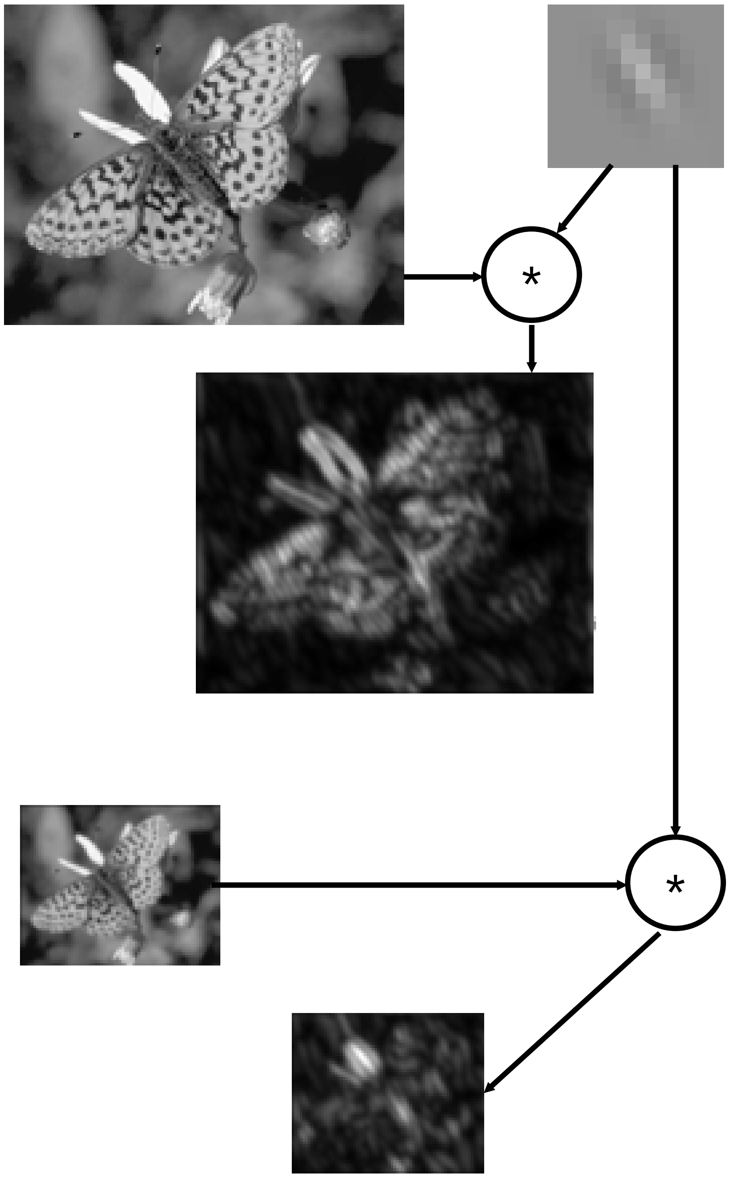


How to filter at multiple scales

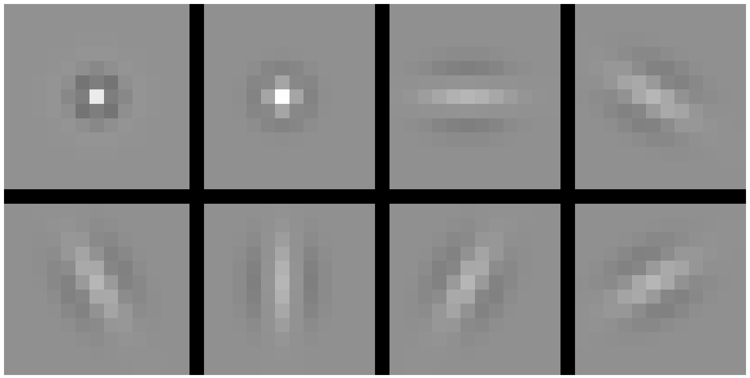
- if two filters have the same shape, the one that has more pixels will detect features of larger scale
- but filtering images with very large filters is expensive
- alternative:
  - keep the filter constant
  - apply it to down-sampled replicas of the image
- the collection of down-sampled images is called a pyramid
- we apply the filter to all levels

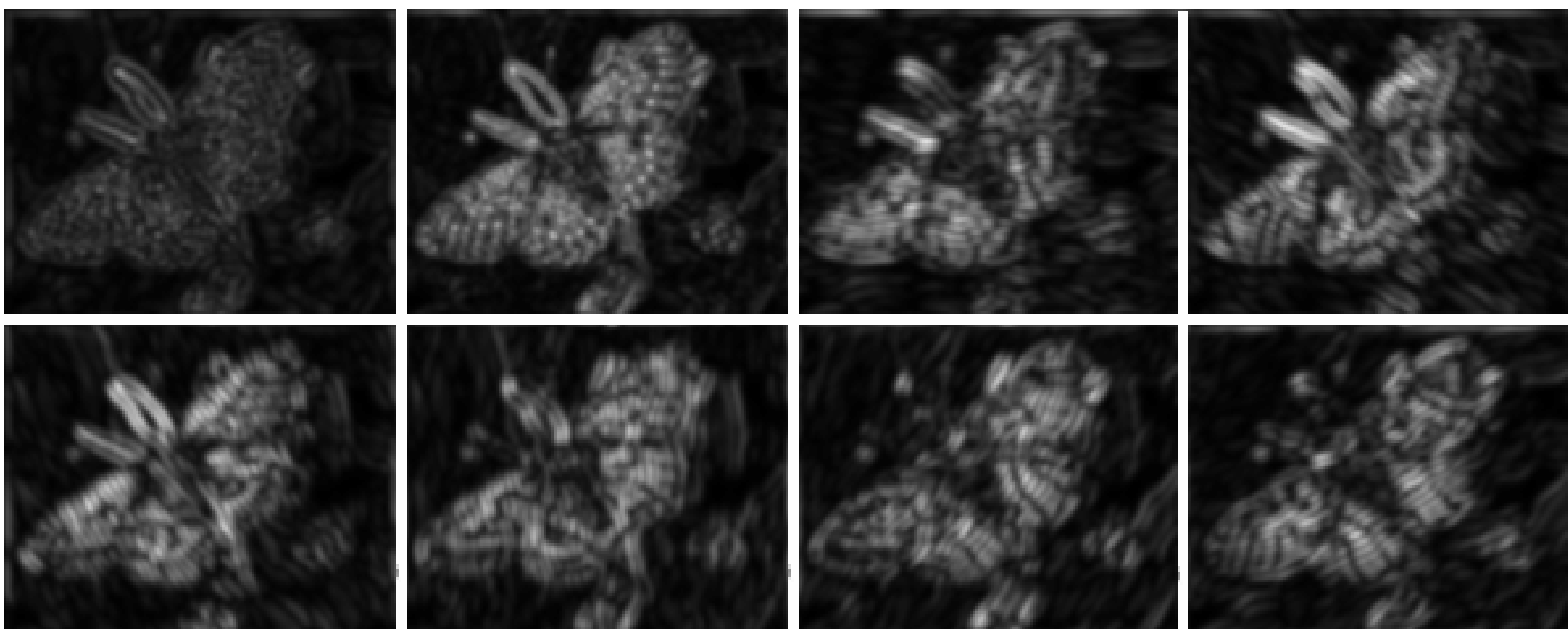
Example

- at high-resolution the filter is quite small
- detects the contours of the antennae, i.e. edges
- at low-resolution it covers a lot more ground
- detects the antennae as "parts"
- note that the stuff which is not oriented like the filter is ignored

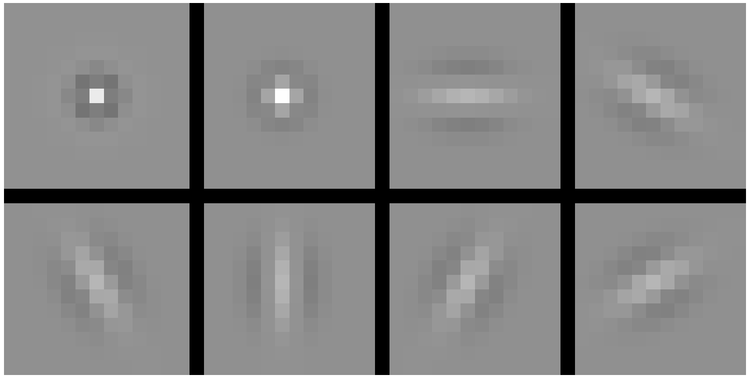


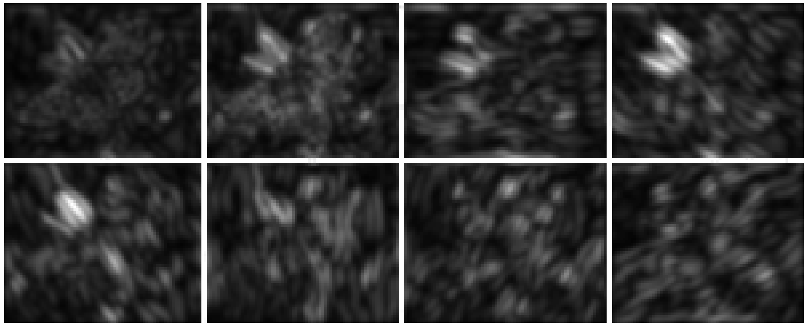
Orientations at small scale





Orientations at large scale

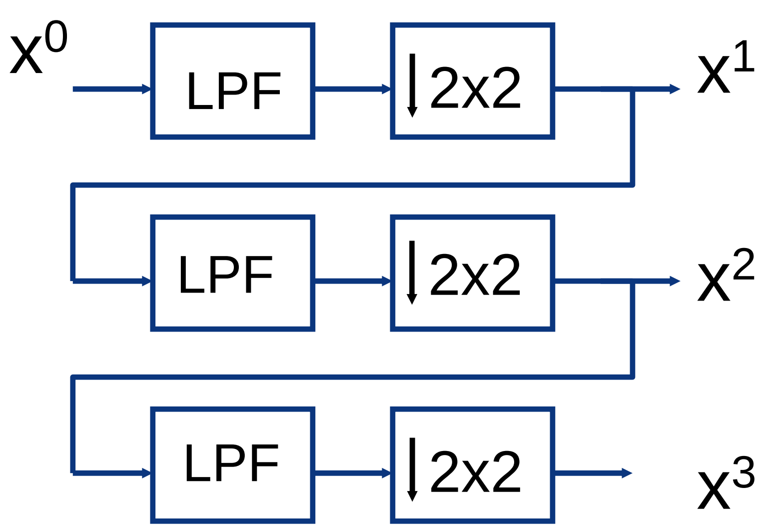


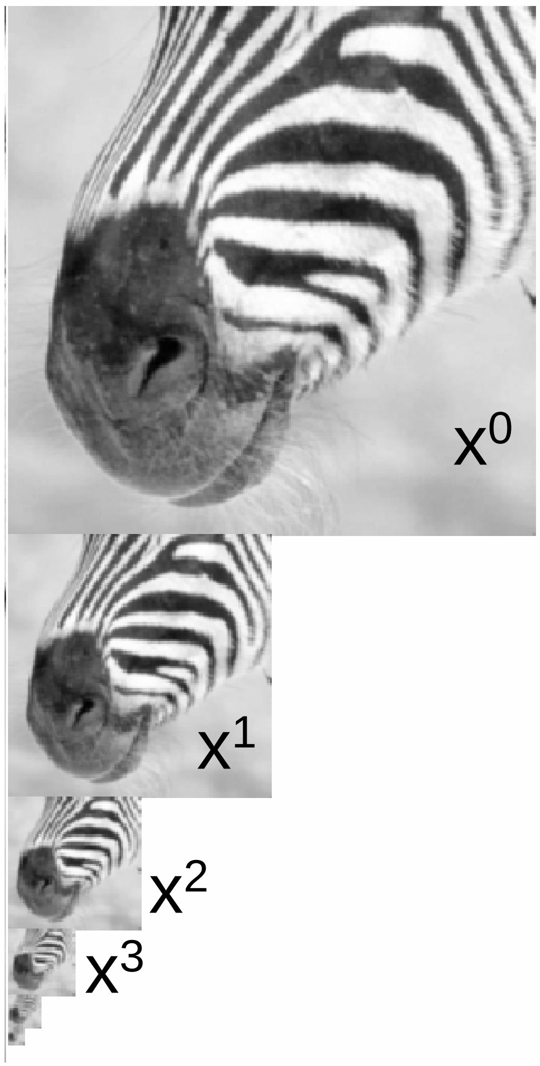


- note that there is significant leakage between orientations (and scales)
- the detectors are not perfect, but provide a workable decomposition
- given this we could say: butterflies have antennae, look for strong response by 4&5, small for the others
- this is an image classifier

Pyramids

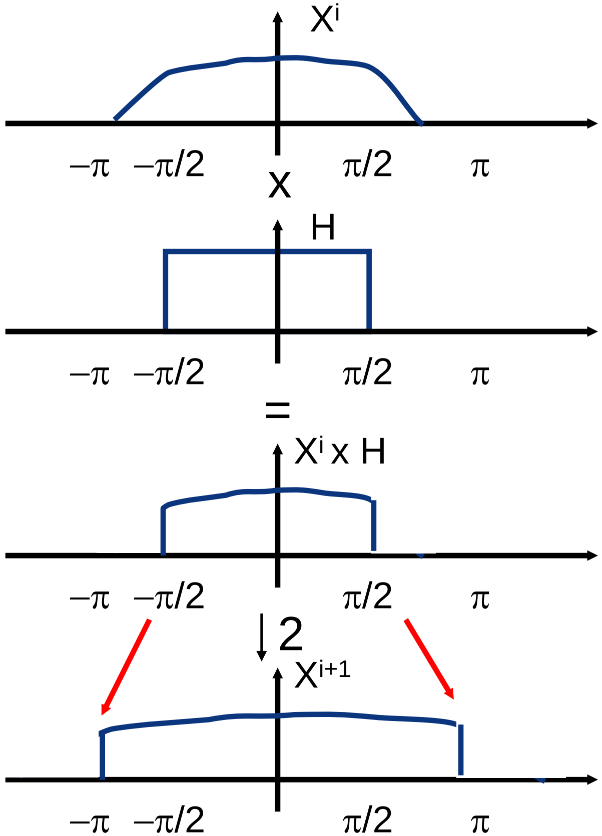
- how do we go about creating a pyramid?
- we want to downsample by two (each direction) at each level
- to avoid aliasing we have to low-pass filter with cut-off $(\pi/2, \pi/2)$.



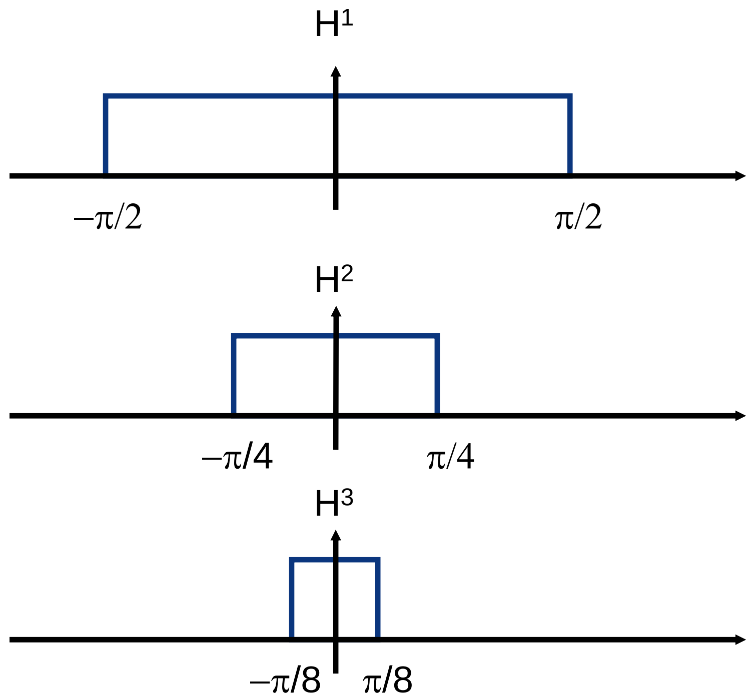


In the frequency domain

- recall: downsamplig expands the spectrum
- each stage:

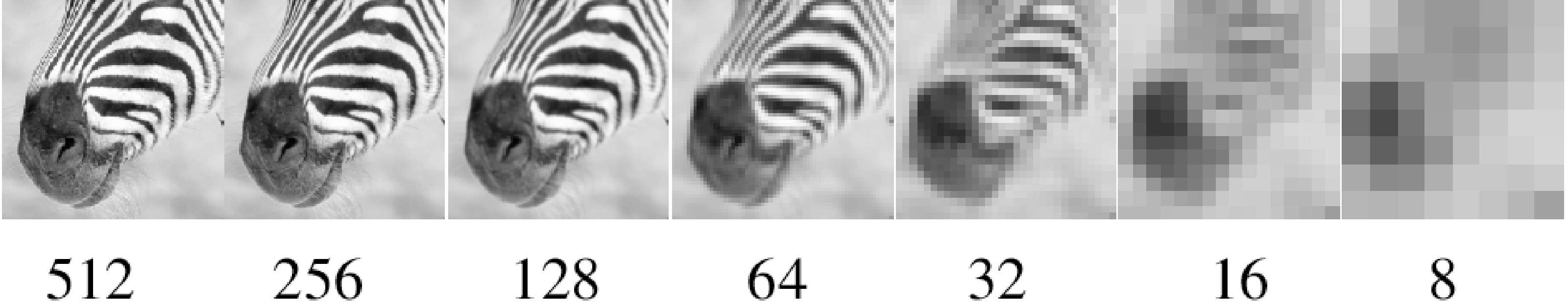


- at full resolution this is equivalent to a sequence of filters



The Gaussian pyramid

- the low pass filter is a Gaussian
- inspired by human vision
  - we have seen that Gaussian type of receptive fields appear in various parts of the brain
- computationally consistent
  - $X_1 \sim N(\mu_1,\sigma_1)$, $X_2 \sim N(\mu_2,\sigma_2)$, independent
  - $Z = X_1 + X_2$,   Z is $N(\mu_1+\mu_2,\sigma_1+\sigma_2)$
  - but $p_Z(z) = p_{X1}(z) * p_{X2}(z)$
- conclusion: gaussian(0,σ)*gaussian(0,σ)=gaussian(0,2σ)
- this is exactly what we need: the n^{th} convolution is low pass filtering with bandwith $\pi/2^n$

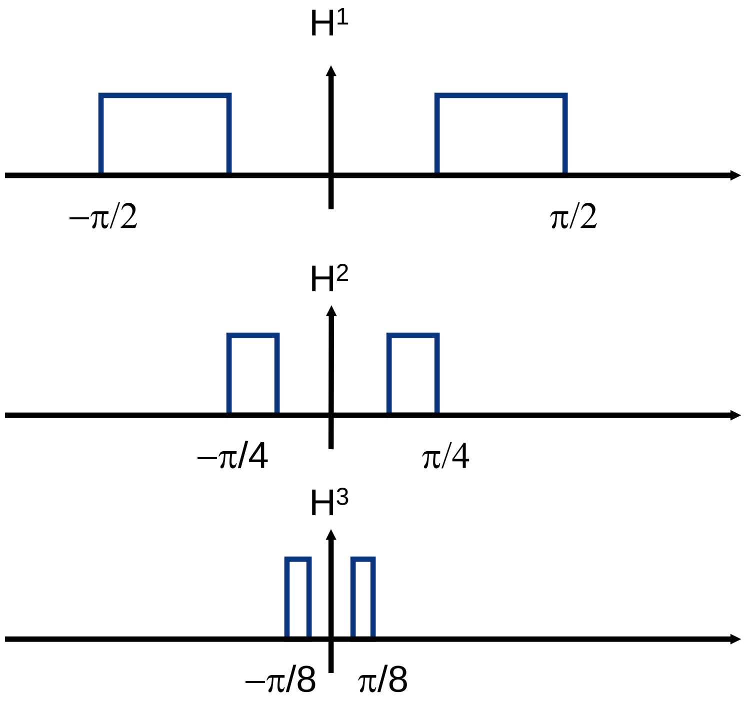


same bar: in the big images is a hair on the zebra's nose; in smaller images, a stripe; in the smallest, the animal's nose

Wavelets

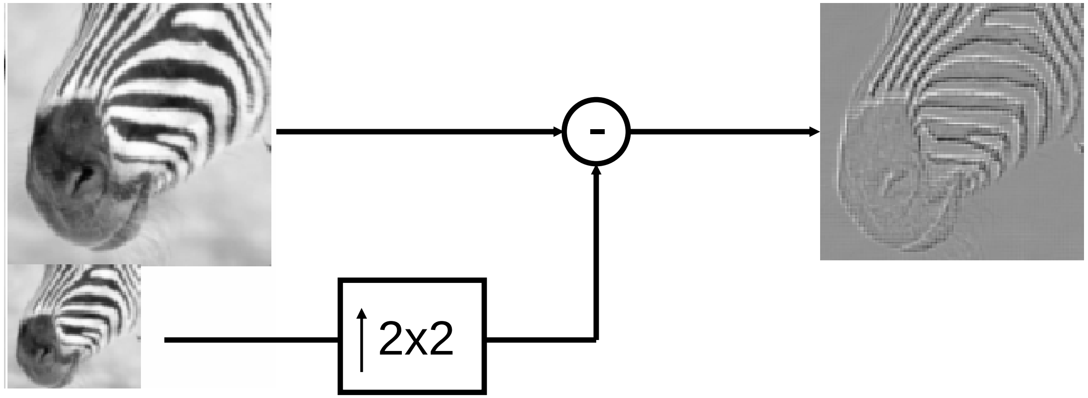
- so far, low frequencies are replicated at all levels
- redundancy problematic for some applications
- wavelets: each filter is bandpass
- this is called a critically sampled pyramid
- no redundancy
- for vision redundancy is sometimes good others not

- at full resolution equivalent to a sequence of filters



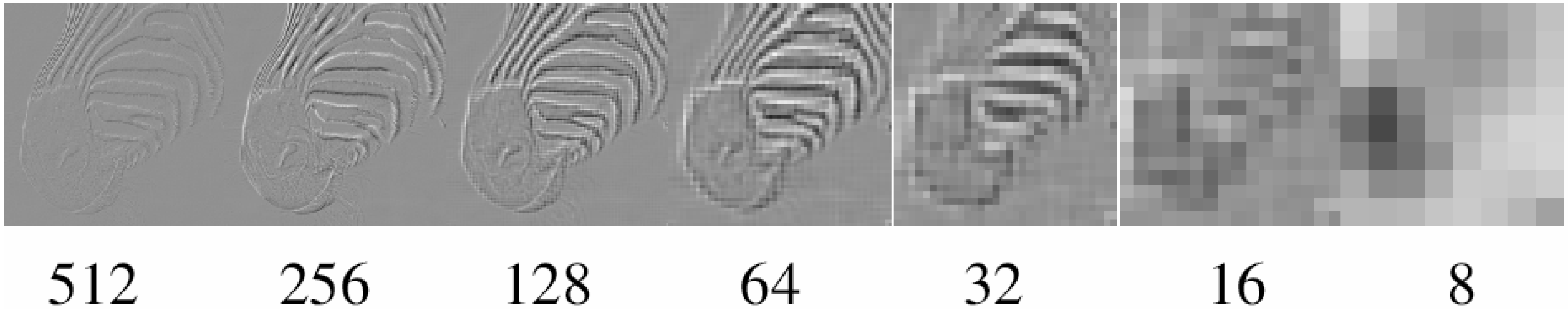
The Laplacian Pyramid

- it is the poor's man version of a wavelet
- to obtain band-pass filtering
  - compute a Gaussian pyramid
  - upsample each level and subtract from its predecessor



  - the same as filtering with a DoG filter, i.e. bandpass
  - lowest resolution is low-pass, other layers have incremental detail

512 256 128 64 32 16 8


512
256
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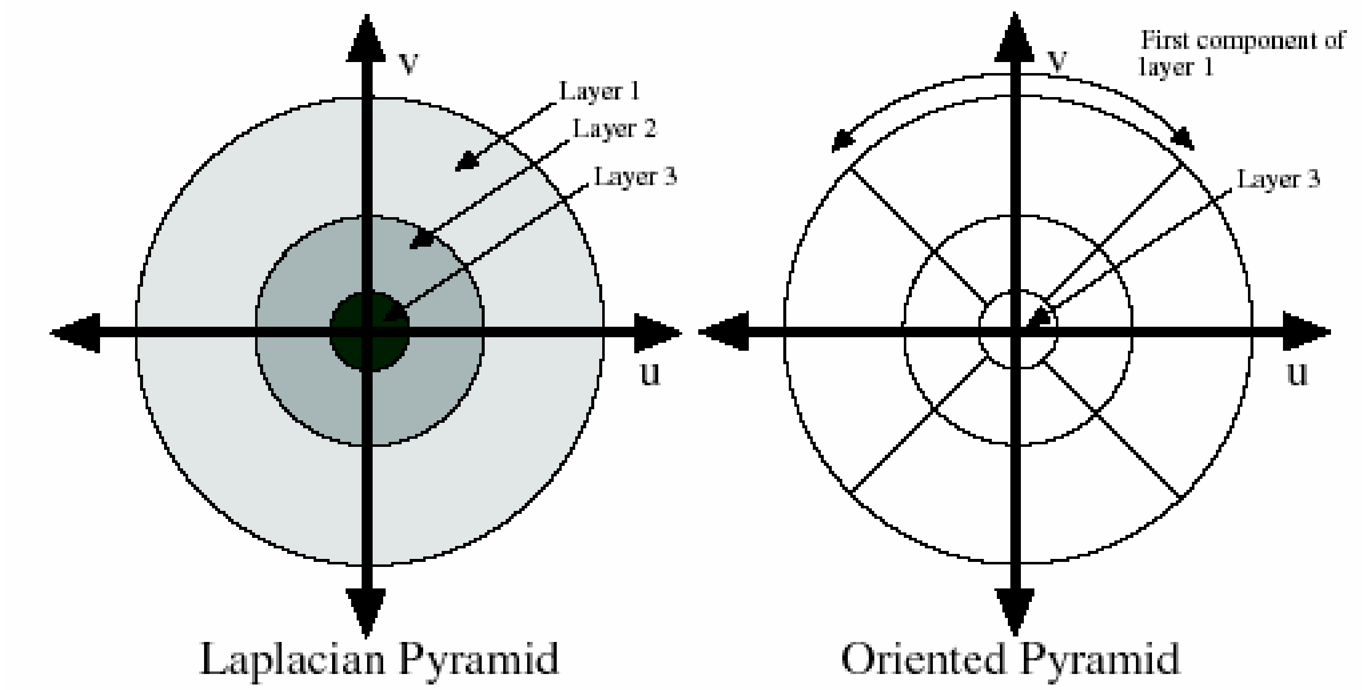
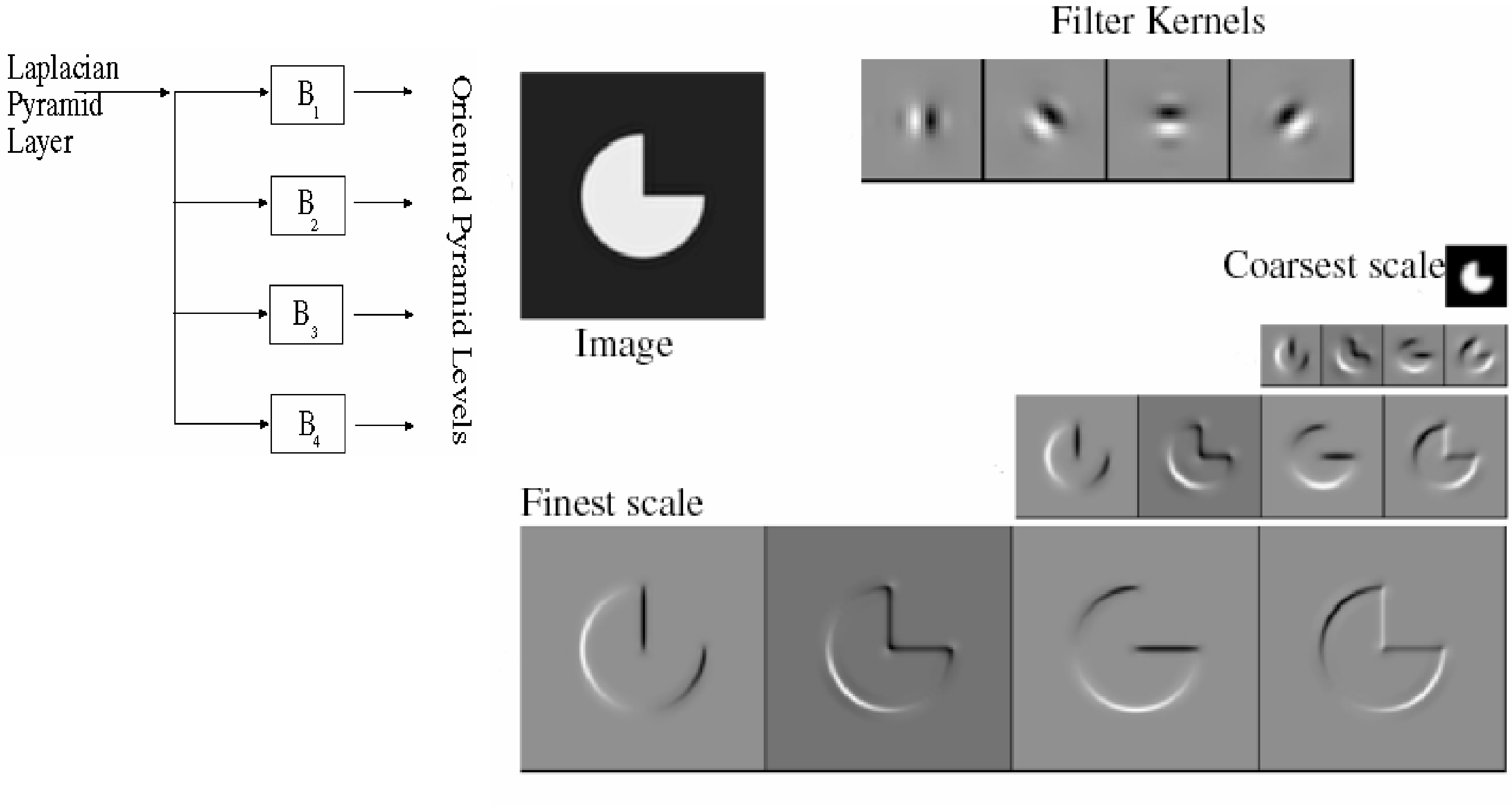
8

Oriented pyramids

- pyramids seen so far do not produce orientation info
- for this need to filter each pyramid level with oriented kernels
- this is an oriented pyramid



Oriented pyramid



- Q: how do we design a filter centered at a certain frequency and with a certain orientation?

Gabor filters

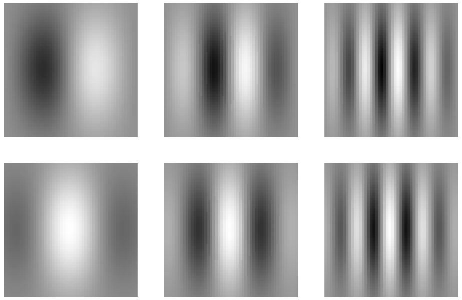
- come in pairs:
  - one recovers symmetric components in a direction,
  - the other recovers antisymmetric components
- definition:

$$G_{sym}(x,y) = \cos(k_x x + k_y y)\exp\left\{-\frac{x^2}{2\sigma_x{}^2} - \frac{y^2}{2\sigma_y{}^2}\right\}$$

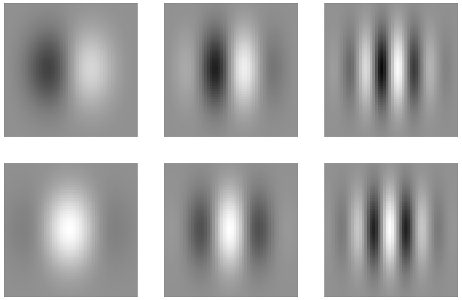
$$G_{sym}(x,y) = \sin(k_x x + k_y y)\exp\left\{-\frac{x^2}{2\sigma_x{}^2} - \frac{y^2}{2\sigma_y{}^2}\right\}$$

- parameters: *(k_x,k_y)* location, *(σ_x,σ_y)* scale

antisymetric



symmetric



In frequency

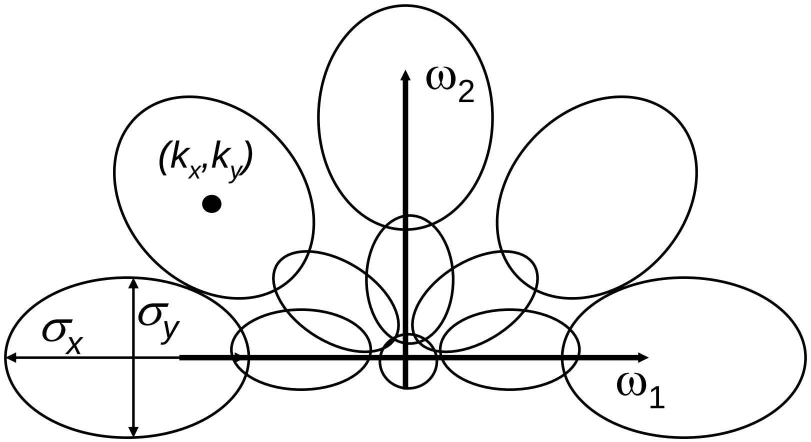
- this is just amplitude modulation of a cosine by a Gaussian

$$G(x) = \cos(k_x x)\exp\left\{-\frac{x^2}{2\sigma_x{}^2}\right\}$$

so

$$G(\varpi) = \exp\left\{-\frac{\sigma_x^2(\varpi - k_x)^2}{2}\right\} + \exp\left\{-\frac{\sigma_x^2(\varpi + k_x)^2}{2}\right\}$$

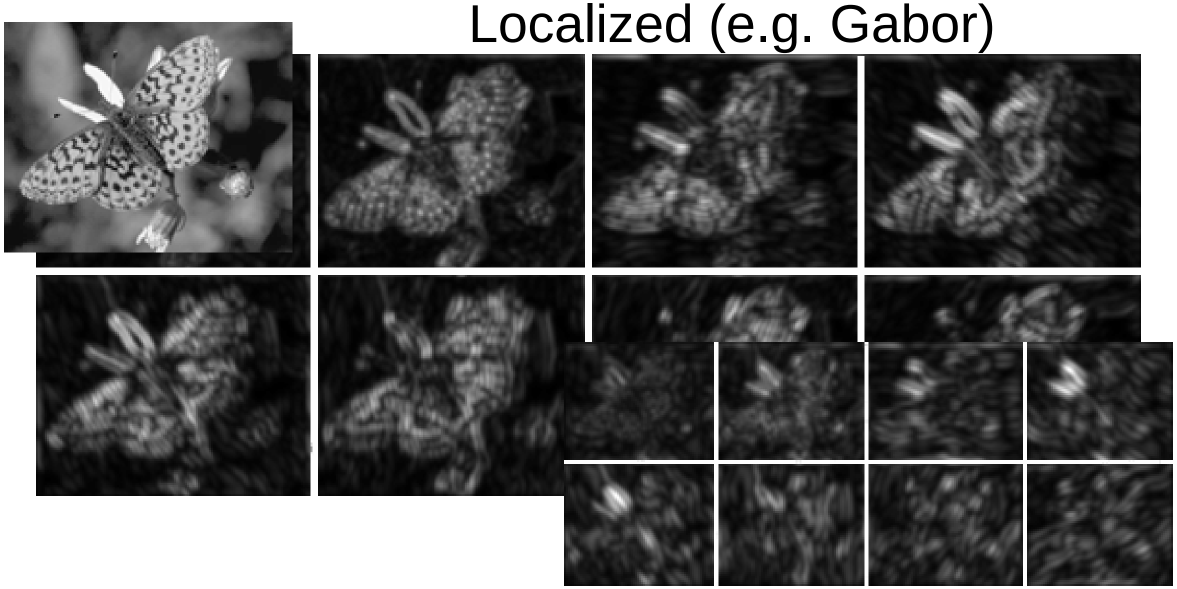
- allows coverage of the frequency spectrum with a set of filters (shown here only ω_2>0)



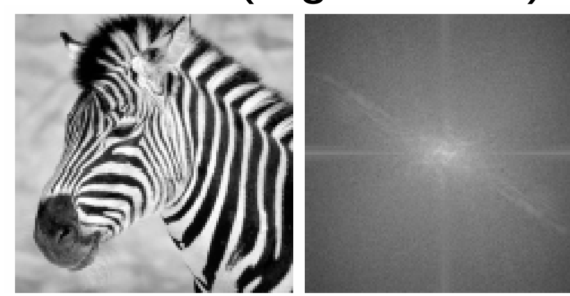
Localization

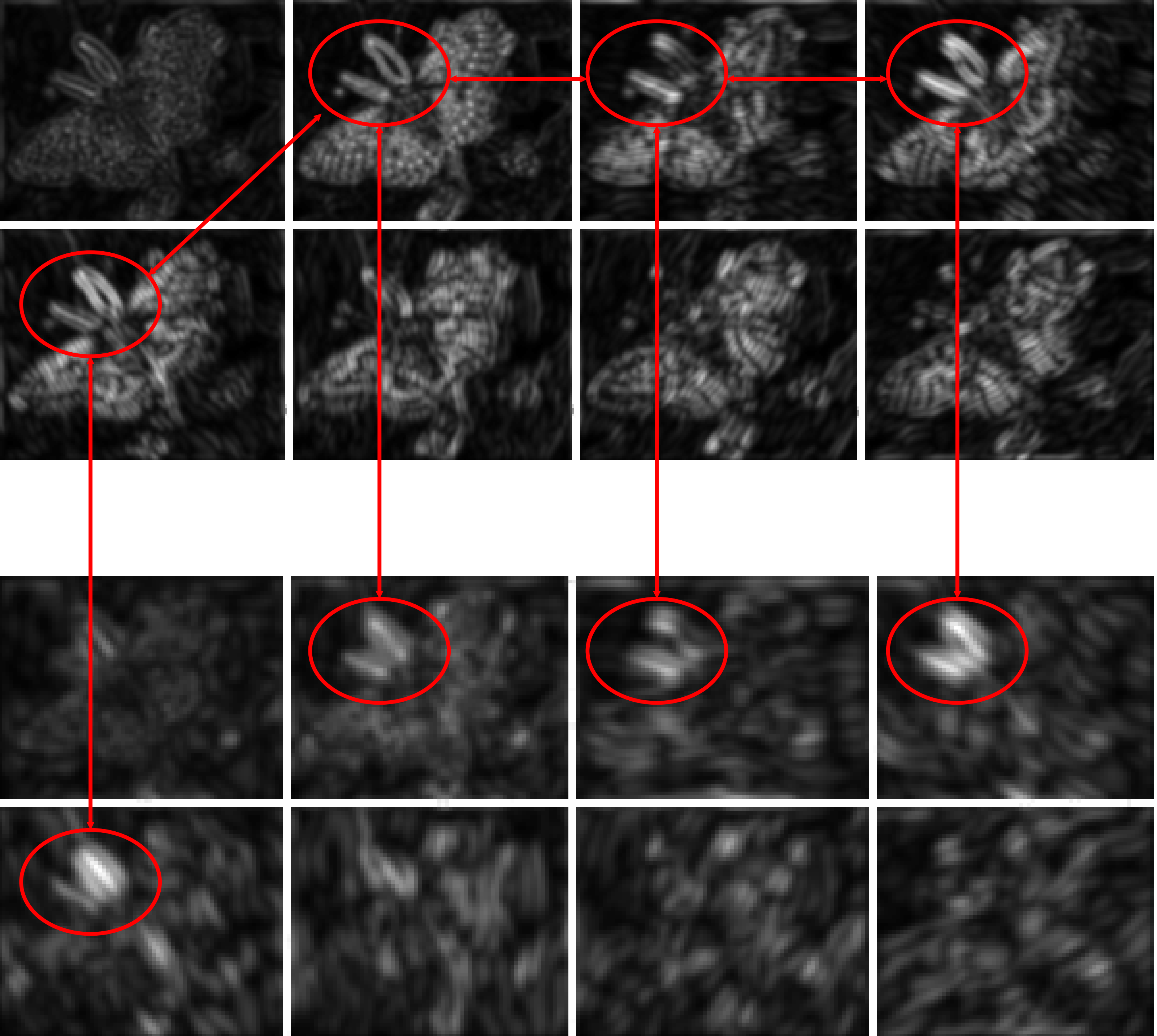
- Gabor localized in space, frequency, and orientation
- decomposition into many frequency "channels"
- contrast with Fourier: non-localized basis functions $e^{j\omega x}$
- localization is important for detailed understanding, e.g. correlations

Localized (e.g. Gabor)



Global (e.g. Fourier)





correlations

Localized representations

- note that Gabor is $G(x,y) = \cos(k_x x + k_y y) w(x,y)$
- where window *w(x,y)* is a Gaussian
- this is what localizes representation in space, cosine (or sine, or $e^{j\omega x}$) is already localized in frequency ($\delta(\omega x)$)
- in fact the localization in frequency gets worse, we go from a Dirac delta to a Gaussian
- once again this is just the uncertainty principle:
  - Fourier: point support in frequency, infinite support in space
  - Gabor: finite (well, close to) support in space, finite support in frequency
- is the Gaussian the only possible window?

Other localized representations

- no. Any low-pass filter will do.
- various wavelets correspond to other choices of window
- note also that if *w(x,y)* is the box filter we get

$$G(x,y) = \cos(k_x x + k_y y) R_{N_1 \times N_2}(x,y)$$

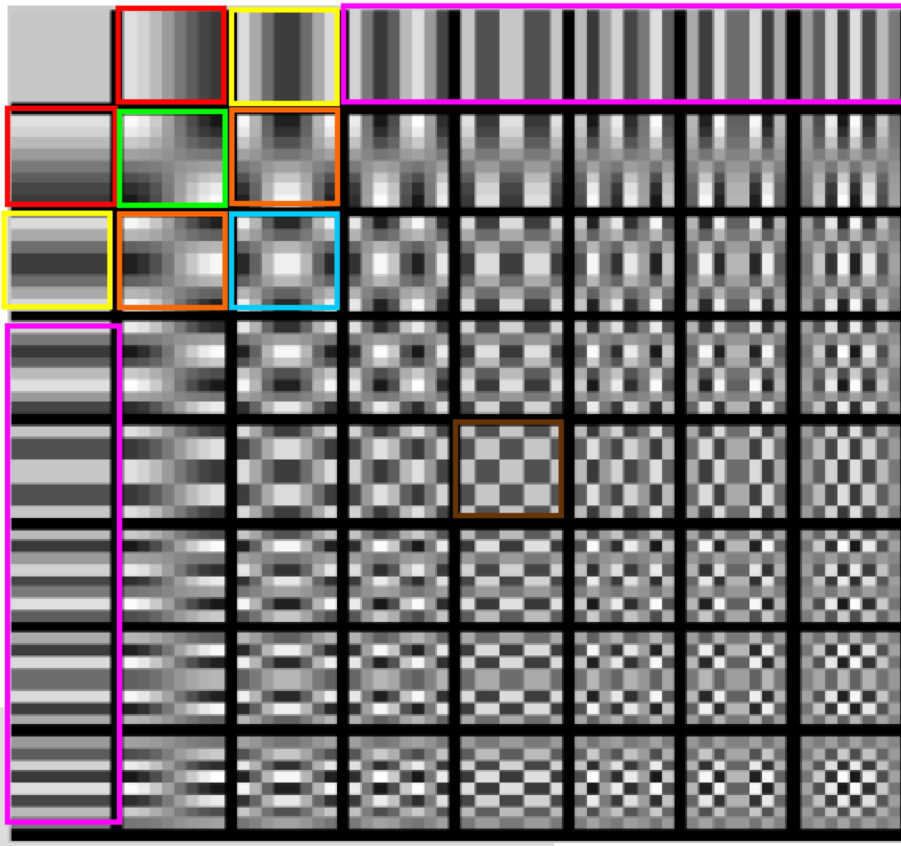
- convolving with these filters is the same as computing the DCT of image blocks
- antisymmetric part corresponds the discrete sine transf.

$$G(x,y) = \sin(k_x x + k_y y) R_{N_1 \times N_2}(x,y)$$

- and is we combine both we get the short-time Fourier transform

$$G(x,y) = e^{j(k_x x + k_y y)} R_{N_1 \times N_2}(x,y)$$

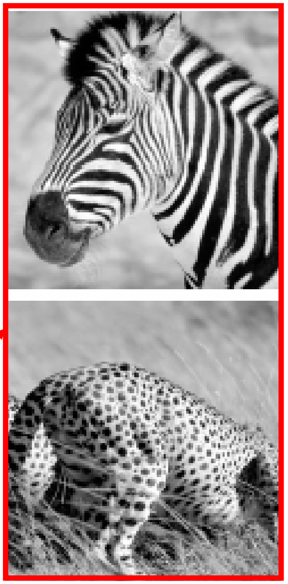
Short-time DCT



- the filters are these
- appealing because
  - real for real images
  - fast, lots of hardware available
- but also because filters detect various attributes that appear relevant
  - vertical/horizontal edges
  - vertical/horizontal bars
  - corners, t-junctions, spot, checkerboards, various flows
- it is also a basis: any function can be reconstructed
- Gabor does not assure that

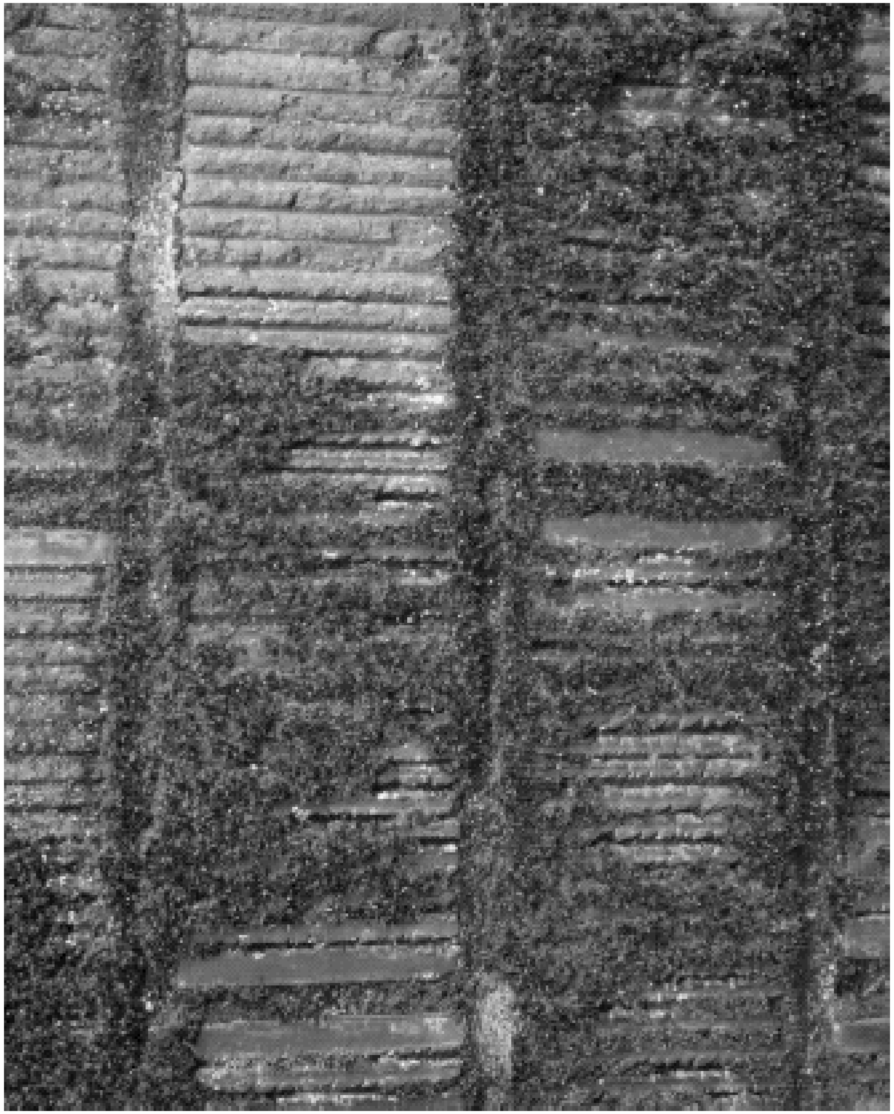
Texture

- we have learned a lot about the importance of localization, scale, and orientation in vision
- but what is this good for in practice?
- one of the major applications is texture analysis/synthesis
- Texture is important for
  - recognition (why is it so easy to recognize a zebra, why is the cheetah not a cat?)
  - segmentation (what are the boundaries between water and grass?)
  - graphics: to synthesize a tiger I need samples of its fur
  - etc.



Representing textures

- but what is a texture?
- I have not heard a good definition yet
- it is one of those things that everyone can recognize, but few can describe, e.g. “like that stuff that X is made of”.
- book: “textures are made up of quite stylised sub-elements, repeated in meaningful ways”
- this is sensible (most of the time) and a workable definition
- anyway, interesting that definition is not easy yet texture gives so much info:
  - e.g. on the next slide it is not clear what “sub-elements” means
  - yet we get plenty of information on geometry, geography, atmospheric conditions, etc.

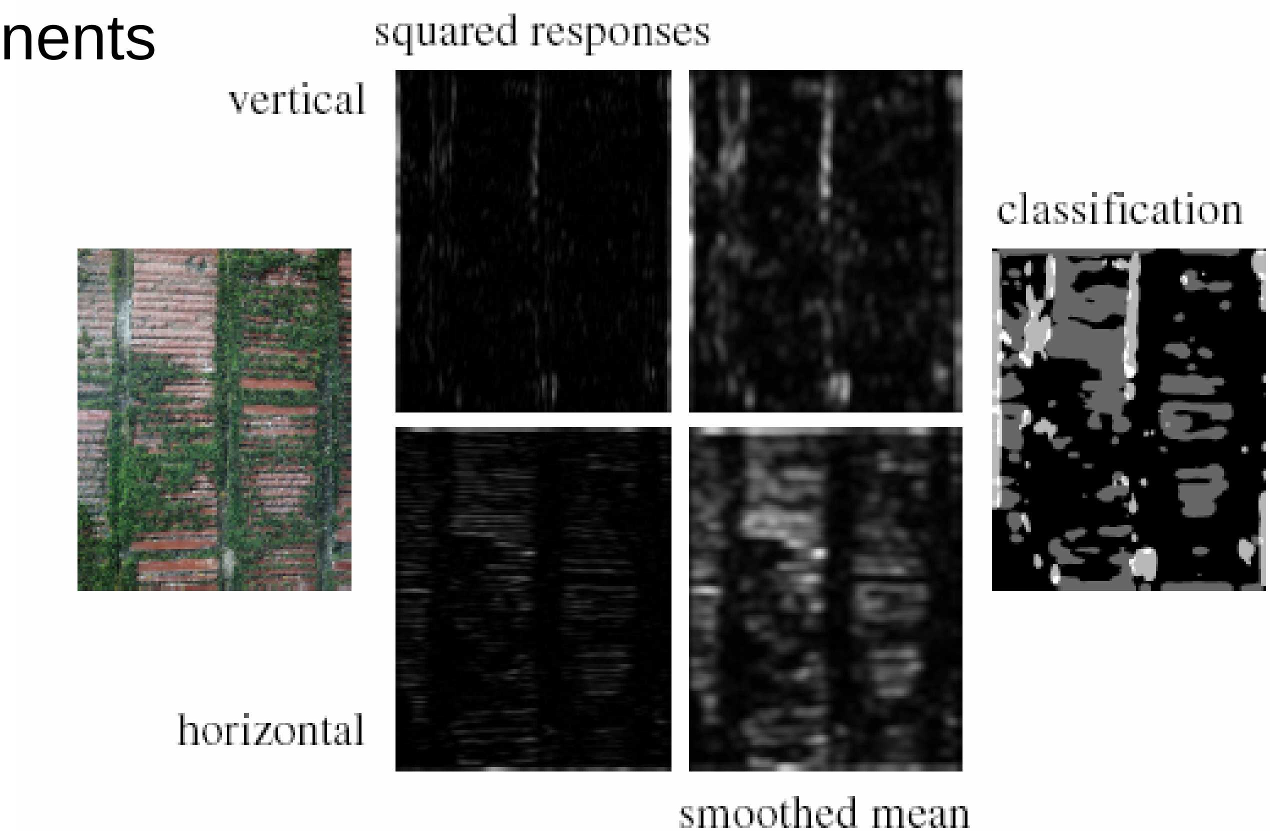


Representing textures

- possible representation:
  - find the sub-elements, and represent their statistics
- but what are the sub-elements, and how do we find them?
  - by applying filters, looking at the magnitude of the response
- what statistics?
  - within reason, the more the merrier.
  - at least, mean and standard deviation
  - better, various probability estimates

Segmentation

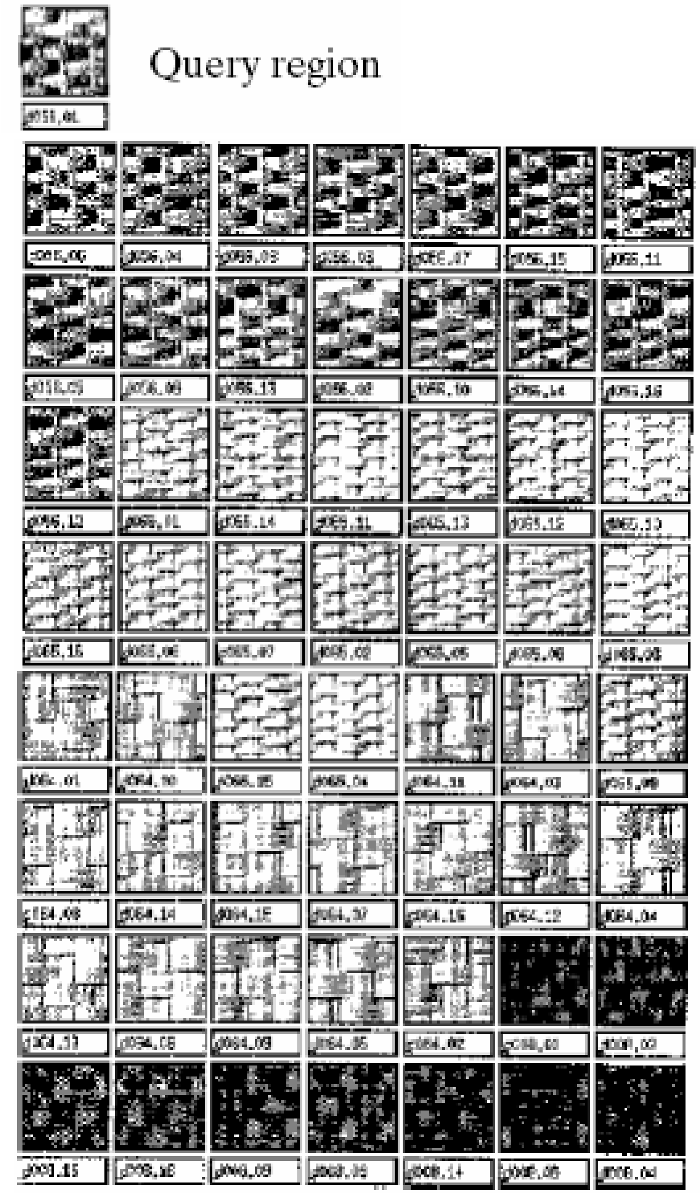
- what are the various components of the scene?
- simple example from the book: two derivatives + square + average over a local window + thresholding
- illustrates segmentation into horizontal/vertical components



Recognition

- what texture is like this?
- example: Gabor decomposition + compute mean and std of each channel + stack in a vector
- each texture in database summarized by one vector t_i
- recognition: find vector t_i closest to query q

$$\min_i \left\| q - t_i \right\|$$



Any questions?