Homework Set Three
ECE 161
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1. Determine whether the following sequences are separable or non-separable. In each case, justify your answer thoroughly (by either *proving* that the sequence is not separable, or determining its decomposition into the product of two sequences).

$$a(n_1, n_2) = \begin{cases} -3, & n_1 = -1, n_2 = 0 \\ 3, & n_1 = 1, n_2 = 0 \\ 3, & n_1 = 0, n_2 = 1 \\ 0, & \text{otherwise} \end{cases}$$

$$b(n_1, n_2) = \begin{cases} 3, & n_1 = 0, n_2 = 0 \\ 9, & n_1 = 0, n_2 = 1 \\ 9, & n_1 = 0, n_2 = 2 \\ -2, & n_1 = 1, n_2 = 0 \\ -6, & n_1 = 1, n_2 = 1 \\ -6, & n_1 = 1, n_2 = 2 \\ 5, & n_1 = 2, n_2 = 0 \\ 15, & n_1 = 2, n_2 = 1 \\ 15, & n_1 = 2, n_2 = 2 \\ 0, & \text{otherwise} \end{cases}$$

$$c(n_1, n_2) = \begin{cases} -3, & n_1 = -1, n_2 = 0 \\ 3, & n_1 = 1, n_2 = 0 \\ 0, & \text{otherwise} \end{cases}$$

$$d(n_1, n_2) = \begin{cases} 3, & n_1 = 0, n_2 = 0 \\ 9, & n_1 = 1, n_2 = 0 \\ 9, & n_1 = 2, n_2 = 0 \\ 5, & n_1 = 0, n_2 = 2 \\ 10, & n_1 = 1, n_2 = 2 \\ 10, & n_1 = 2, n_2 = 2 \\ 0, & \text{otherwise} \end{cases}$$

2. Let $h[n_1, n_2, n_3]$ be a separable 3D sequence of $M \times M \times M$ points which can be expressed as

$$h[n_1, n_2, n_3] = a[n_1]b[n_2]c[n_3]$$

a) the Fourier transform of $h[n_1, n_2, n_3]$ is defined as

$$H(\omega_1, \omega_2, \omega_3) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \sum_{n_3=-\infty}^{\infty} h[n_1, n_2, n_3] e^{-j\omega_1 n_1} e^{-j\omega_2 n_2} e^{-j\omega_3 n_3}.$$

Show that $H(\omega_1, \omega_2, \omega_3)$ is a separable function that can be written in the form $A(\omega_1)B(\omega_2)C(\omega_3)$. What are $A(\omega_1)$, $B(\omega_2)$, and $C(\omega_3)$?

b) we wish to filter an input sequence $x[n_1, n_2, n_3]$ of $N \times N \times N$ points using an LSI system with impulse response $h[n_1, n_2, n_3]$. Develop a computationally efficient way to obtain the output $y[n_1, n_2, n_3]$.

c) how does your method compare to direct evaluation of the convolution sum for each output point when $N = 256$ and $M = 5$?

3. Consider the sequences

$$x(n_1, n_2) = \begin{cases} 6, & n_1 = 0, n_2 = 2 \\ 4, & n_1 = 0, n_2 = 1 \\ 2, & n_1 = 1, n_2 = 0 \\ 3, & n_1 = 2, n_2 = 0 \\ 0, & \text{otherwise} \end{cases}$$

and

$$h(n_1, n_2) = \begin{cases} 1, & n_1 = 0, n_2 = 0 \\ 1, & n_1 = 0, n_2 = 1 \\ 1, & n_1 = 1, n_2 = 0 \\ 1, & n_1 = 1, n_2 = 1 \\ 0, & \text{otherwise} \end{cases}$$

a) compute the convolution $x(n_1, n_2) * h(n_1, n_2)$ by the two ways discussed in class. In each of the cases, include an intermediate number of steps sufficient to leave no ambiguity as to which method was used.

b) determine the DSFT $X(\omega_1, \omega_2)$ of $x(n_1, n_2)$.

c) determine the inverse DSFT of $Z(\omega_1, \omega_2) = X(\omega_1, \omega_2)e^{-j(\omega_1 + 2\omega_2)}$

d) compute

$$\int_{\omega_1=-\pi}^{\pi} \int_{\omega_2=-\pi}^{\pi} |Z(\omega_1, \omega_2)|^2 d\omega_1 d\omega_2$$

4. In class we will see that the following filters

$$\begin{array}{cc} 0 & 0 \\ -1 & 1 \\ 0 & 0 \end{array} \quad \text{and} \quad \begin{array}{cc} -1 & 1 \\ -1 & 1 \\ -1 & 1 \end{array}$$

play an important role in edge detection. Assuming that the origin ($n_1 = n_2 = 0$) is the element in the 2^{nd} row and 1^{st} column, answer the following.

a) what is the DSFT of the edge detection filter in each of the cases?

b) both filters are separable. What are the 1-D sequences into which they can be decomposed? What are their DTFTs (1D discrete time Fourier transforms)?

c) for each filter, hand-in a surface plot of the 2D DSFT and a plot of the 1D DTFTs of each of the composing sequences.

d) explain the differences between the two surface plots in terms of the differences between the composing two 1D sequences.

e) which filter would you use to filter a noisy image? Why?

5. (MATLAB) In this problem we study the relationships between image patterns and their DSFTs.

a) Download the test image patches, `test1.jpg, test2.jpg, test3.jpg and test4.jpg`, compute their DSFT (you can use the function `fft2(.)`, which produces a discrete version of the DFST), and plot the frequency spectrum of each image patch. You only need to plot the absolute value of the spectrum. Use a logarithmic scale (i.e. plot the log of the magnitude) to make the details visible. Explain the shape of the spectrum of each of the patterns, in particular frequency patterns that you would not expect from inspection of the image (recall that some of the patterns are sinusoids). If you find some patterns troubling, inspect the image closely (e.g. by opening it up in an image viewer and scaling it up).

b) For the remaining image, `Spectra.jpg`, take the DFST and plot the frequency spectrum, using the same procedure as above. What are the patterns that you see in the spectrum? Which pattern corresponds to which component (or region) of the image. Carefully explain all the image vs. spectrum matches, and explain as many features of the spectrum as you can.