

Homework Set Six
ECE 161
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1. In class, we saw that an affine transformation is characterized by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}.$$

a) Assume that we computed the motion between two images and obtained the motion vector field $(u(x, y), v(x, y))$, where $u(x, y)$ and $v(x, y)$ are the horizontal and vertical components of the displacement of pixel (x, y) from the first to the second image. Because motion estimation is a noisy process, the motion vectors do not satisfy exactly the affine equation. What is the least squares estimate of the parameters (a, b, c, d, e) based on the measured motion vectors?

b) Suppose that your least squares parameter estimate satisfies $b = -c$, and that the motion consisted of a rotation by θ radians, followed by scaling, and translation. Determine expressions for the amounts of rotation, scaling, and translation as a function of the affine parameters.

2. Consider the image `lena.jpg` and the following transformations

Transformation	parameters
Rotation	$\frac{\pi}{4}$
Translation	(10, 15) pixels
Scaling	(2, 3) times

For translation and scaling, the first value is relative to the x , and the second to the y coordinate. Suppose that when an unknown image was subject to the transformations, the result was `lena`.

a) For each transformation, hand-in a **quiver** plot of the associated motion vector field (note: feel free to show only a subset of the motion vectors if that makes the plot more clear).

b) For each transformation hand in a surface plot of the components $u(x, y)$ and $v(x, y)$ of the associated motion vector field. What are these surfaces?

c) For each transformation, hand in a plot of the image that was warped to produce `lena`. (note: for each pixel determine the corresponding pixel in `lena` and then use `interp2()`).

3. In class, we have shown that when the goal is to find the motion vector (u, v) that minimizes the error

$$\sum_{x,y \in R} [I(x-u, y-v, t) - I(x, y, t+1)]^2$$

for the motion of the pixels in window R between images t and $t+1$, a differential solution will lead to a set of equations of the form

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum_{x,y \in R} I_x^2(x, y) & \sum_{x,y \in R} I_x(x, y) I_y(x, y) \\ \sum_{x,y \in R} I_x(x, y) I_y(x, y) & \sum_{x,y \in R} I_y^2(x, y) \end{bmatrix}^{-1} \begin{bmatrix} \sum_{x,y \in R} I_x(x, y) I_t(x, y) \\ \sum_{x,y \in R} I_y(x, y) I_t(x, y) \end{bmatrix}$$

where I_x , I_y , and I_t are the partial derivatives in the x , y , and t dimensions. Suppose that instead of just translation we need to recover the six parameters of an affine transformation, i.e. we want to minimize the error

$$\sum_{x,y \in R} [I(x - (ax + by + e), y - (cx + dy + f), t) - I(x, y, t + 1)]^2.$$

Derive the system of equations whose solution leads to the optimal parameter vector $(a, b, c, d, e, f)^T$.

4. (MATLAB) Write a function to compute the Gaussian pyramid of a given image. Use the filter given by the following lines of MATLAB code

```
a = 0.5; b = 0.25; c = 0.25-a/2; w = [c b a b c]';
```

a) Hand-in a plot of the 5 images obtained with a 5-level Gaussian pyramid decomposition of the image `lena.jpg`.

b) Modify your code to compute the Laplacian pyramid. Hand-in a plot of the 5 images obtained with a 5-level Laplacian pyramid decomposition of the image `lena.jpg`.

c) Using `randn()` add Gaussian noise of zero mean and standard deviation $\sigma = 10$ to the image. Recompute the Laplacian pyramid for the noisy image. What are the differences with respect to **c)**? Can you comment on the differences in the distribution of signal energy, through the various frequency bands, between 1) edges and 2) independent Gaussian noise?