**Homework Set Six**
ECE 161
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**1.** In class, we saw that an affine transformation is characterized by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}.$$

**a)** Assume that we computed the motion between two images and obtained the motion vector field $(u(x, y), v(x, y))$, where $u(x, y)$ and $v(x, y)$ are the horizontal and vertical components of the displacement of pixel $(x, y)$ from the first to the second image. Because motion estimation is a noisy process, the motion vectors do not satisfy exactly the affine equation. What is the least squares estimate of the parameters $(a, b, c, d, e, f)$ based on the measured motion vectors?

**b)** Suppose that your least squares parameter estimate satisfies $b = -c$, and that the motion consisted of a rotation by $\theta$ radians, followed by scaling, and translation. Determine expressions for the amounts of rotation, scaling, and translation as a function of the affine parameters.

**2.** Consider the image `lena.jpg` and the following transformations

| Transformation | parameters |
|---|---|
| Rotation | $\frac{\pi}{4}$ |
| Translation | $(10, 15)$ pixels |
| Scaling | $(2, 3)$ times |

For translation and scaling, the first value is relative to the $x$, and the second to the $y$ coordinate. Suppose that when an unknown image was subject to the transformations, the result was `lena`.

**a)** For each transformation, hand-in a `quiver` plot of the associated motion vector field (note: feel free to show only a subset of the motion vectors if that makes the plot more clear).

**b)** For each transformation hand in a surface plot of the components $u(x, y)$ and $v(x, y)$ of the associated motion vector field. What are these surfaces?

**c)** For each transformation, hand in a plot of the image that was warped to produce `lena`. (note: for each pixel determine the corresponding pixel in `lena` and then use `interp2()`).

**3.** In class, we have shown that when the goal is to find the motion vector $(u, v)$ that minimizes the error

$$\sum_{x,y \in R} [I(x-u, y-v, t) - I(x, y, t+1)]^2$$

for the motion of the pixels in window $R$ between images $t$ and $t+1$, a differential solution will lead to a set of equations of the form

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \sum_{x,y \in R} I_x^2(x,y) & \sum_{x,y \in R} I_x(x,y) I_y(x,y) \\ \sum_{x,y \in R} I_x(x,y) I_y(x,y) & I_y^2(x,y) \end{bmatrix}^{-1} \begin{bmatrix} \sum_{x,y \in R} I_x(x,y) I_t(x,y) \\ \sum_{x,y \in R} I_y(x,y) I_t(x,y) \end{bmatrix}$$

where $I_x$, $I_y$, and $I_t$ are the partial derivatives in the $x$, $y$, and $t$ dimensions. Suppose that instead of just translation we need to recover the six parameters of an affine transformation, i.e. we want to minimize the error

$$\sum_{x,y \in R} [I(x - (ax + by + e), y - (cx + dy + f), t) - I(x, y, t+1)]^2.$$

Derive the system of equations whose solution leads to the optimal parameter vector $(a, b, c, d, e, f)^T$.

**4. (MATLAB)** Write a function to compute the Gaussian pyramid of a given image. Use the filter given by the following lines of MATLAB code

```
a = 0.4; b = 0.25; c = 0.25-a/2; w = [c b a b c]';
```

**a)** Hand-in a plot of the 5 images obtained with a 5-level Gaussian pyramid decomposition of the image `lena.jpg`.

**b)** Modify your code to compute the Laplacian pyramid. Hand-in a plot of the 5 images obtained with a 5-level Laplacian pyramid decomposition of the image `lena.jpg`.

**c)** Using `randn()` add Gaussian noise of zero mean and standard deviation $\sigma = 10$ to the image. Recompute the Laplacian pyramid for the noisy image. What are the differences with respect to **c)**? Can you comment on the differences in the distribution of signal energy, through the various frequency bands, between 1) edges and 2) independent Gaussian noise?