Radiometry

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Light

- Last class: geometry of image formation
- pinhole camera:
  - point \((x, y, z)\) in 3D scene projected into image pixel of coordinates \((x', y')\)
  - according to the perspective projection equation:
Perspective projection

• the inverse dependence on depth (Z)
  – causes objects to shrink with distance

• while pinhole is a good mathematical model

• in practice, cannot really use it
  – not enough light for good pictures
Lenses

• the basic idea is:
  – let's make the aperture bigger so that we can have many rays of light into the camera
  
  – to avoid blurring we need to concentrate all the rays that start in the same 3D point
  
  – so that they end up on the same image plane point
Lenses

- fundamental relation

\[ \frac{1}{d_1} \approx \frac{1}{n_1} \left( \frac{n_2 - n_1}{R} - \frac{n_2}{d_2} \right) \]

  - note that it does not depend on the vertical position of \( P \)
  - we can show that it holds for all rays that start in the plane of \( P \)
Lenses

• note that, in general,
  – can only have in focus objects that are in a certain depth range
  – this is why background is sometimes out of focus

  – by controlling the focus you are effectively changing the plane of the rays that converge on the image plane without blur

• for math simplicity, we will work with pinhole model!
Light

• today: what is the pixel brightness or image intensity?
• clearly, depends on three factors:
  – lighting of the scene
  – the reflectance properties of the material
  – various angles
Radiance

• to study light

• first important concept is radiance
  – appropriate unit for measuring distribution of light

• **Definition:** radiance is
  – power (energy/unit time) traveling at \( x \) in direction \( \mathbf{V} \)
  – per unit area perpendicular to \( \mathbf{V} \)
  – per unit solid angle

• measured in
  – watts/square meter \( \times \) steradian (\( w \times m^{-2} \times sr^{-1} \))
  – (steradian = radian squared)

• it follows that it is a function of a point \( x \) and a direction \( \mathbf{V} \)
Radiance

• important property
  – in a lossless medium (e.g. air)
  – whatever radiance is emitted by the object at \( P_o \)
  – is the radiance that is received by the image at \( P_i \)

• “in a lossless medium radiance is constant along straight lines”
Light

- the next question is:
  - what is the relation between
    - the illumination that reaches the object
    - and the reflected light?

- this is measured by the bidirectional reflectance distribution function (BRDF)
BRDF

- is the ratio of energy in outgoing direction ($V_o$) to incoming direction ($V_i$)
  $$\rho_{bd} (P, V_i, V_o)$$

- important property (Helmoltz reciprocity)
  - BRDF is symmetric
    $$\rho_{bd} (P, V_i, V_o) = \rho_{bd} (P, V_o, V_i)$$

- but we can do even simpler than this
  - for Lambertian surfaces, the BRDF does not depend direction at all
  - they reflect light equally in all directions
    $$\rho_{bd} (P, V_i, V_o) = \rho (P)$$
Albedo

• in this case the surface is described by its albedo

\[ \rho(P) \]

• note that
  – most surfaces are not Lambertian,
  – but the Lambertian assumption makes the equations a lot easier
  – commonly used in practice, even though Lambertian objects do not appear realistic (not good enough for graphics)
Albedo

• in laymen’s terms:
  – albedo is percentage of light reflected by an object
  – it depends on the color and material properties of the object
  – light colors reflect more light (why you should wear white in the desert)
  – this turns out to have major consequences for object temperature
  – e.g., it is one of the main justifications for global warming
Albedo and global warming

• snow turns out to have the largest possible albedo, and reflects almost 100% of the light
• most other objects absorb light, and heat up
Albedo and global warming

• by reflecting most of the incident light
• the polar cap cools off the planet
• as ice melts, less light is reflected
• the planet warms up, more ice melts, etc.
• this is one of the main reasons for global warming

1 Light colored ice reflects back the Sun’s energy efficiently.
2 Exposed land is darker colored and absorbs more energy.
3 As the ice melts, more land is exposed. This absorbs more heat, melting more ice.
4 The altitude of the melting ice is reduced so it becomes harder for new ice to form.
Albedo and parking lots

• these day, this effect is taken very seriously
• it turns out that increasing reflections is useful in many other ways
• for example, it can save a lot of lighting (energy)
• an example of how paving parking lots with pavement of higher reflectance can make a difference
Angles

• is the object **albedo the only factor that matters** for how much light it reflects?
• **no**, the **angle** at which the light is incident also matters
Light

- this is easy to see
- consider the following experiment

![Diagram showing a light source and a sheet of paper with lots of reflected light on one side and no reflection on the other.]

- energy absorbed by object depends on its surface area
- this varies with the incident angle
- concept that captures this dependence on angles is that of foreshortening
Light

- **foreshortening**: very important concept
  - tilted surface looks smaller than when seen at 90°
  - best understood by example
  - if I show you a tilted person it looks smaller than when you view you at 90°
Light

• what is the foreshortened area for a patch of area $dA$?

• it depends on the angle $\theta$ between
  – the normal to the patch
  – viewing direction

• for a known area $dA$ we can actually compute the foreshortening factor
Light

• this is easy for a simple case

• foreshortened area is

\[ dA' = dl_1' dl_2' = dl_1 dl_2 \cos \theta = dA \cos \theta \]

• it can be shown that this holds for a patch of any shape.
  
  – **foreshortened area = area \times \cos (angle between viewing direction and surface normal)**
Lambertian surfaces

• putting everything together,
  – we have an equation for the light reflected by an object
  – assuming surface reflects equally in all directions (Lambertian)
  – outgoing radiance is

\[ L(P, V_o) = \rho(P) L(P, V_i) \cos \theta_i, \forall V_o \]

– “light reflected at point \( P \) in direction \( V_o = \) albedo at \( P \) x incident light from direction \( V_i \) x cos (normal, incident)”

– note that
  • it holds for any outgoing direction \( V_o \)
  • is a function of \( V_i \)
  • if there are multiple incoming directions, we have to integrate over \( V_i \)
Lambertian surfaces

- this allows us to propagate light throughout a scene
Lambertian surfaces

- using constancy of radiance along straight lines

\[ L(P_1, V_1) = L(P_0, V_1) \]

\[ L(P_1, V_2) = L(P_1, V_1) \rho(P_1) \cos \theta_2, \forall V_2 \]

\[ \Downarrow \]

\[ L(P_1, V_2) = L(P_0, V_1) \rho(P_1) \cos \theta_2, \forall V_2 \]
Lambertian surfaces

• using reflection equation again

\[ L(P_0, V_1) = L(P_0, V_0) \rho(P_0) \cos \theta_1, \forall V_1 \]

\[ L(P_1, V_2) = L(P_0, V_1) \rho(P_1) \cos \theta_2, \forall V_2 \]

\[ L(P_1, V_2) = L(P_0, V_0) \rho(P_1) \rho(P_0) \cos \theta_2 \times \cos \theta_1 \]
Lambertian surfaces

- and we have a rule for any number of bounces

\[
L(P_n, V) = L(P_0, V_0) \left[ \prod_{i=0}^{n} \rho(P_i) \right] \left[ \prod_{i=1}^{n+1} \cos \theta_i \right], \forall V
\]
Lambertian surfaces

- note that on

\[
L(P_n, V) = L(P_0, V_0) \left[ \prod_{i=0}^{n} \rho(P_i) \right] \left[ \prod_{i=1}^{n+1} \cos \theta_i \right], \forall V
\]

- unless all cosines are close to 1
- their product goes to zero quickly
- e.g. see decay of \(\cos^n(\theta)\) with \(n\)
- this means that only light that arrives frontally to all the bounces gets propagated very far
- such an alignment is very unlikely
- we don’t really have to worry about many bounces
- the process becomes tractable
Lambertian surfaces

• on the other hand,
  – there are still various single-bounce paths
  – e.g. each source has a single bounce path to each non-shaded object
  – to deal with this we need to know more about light sources
Light sources

• most common model is “point source at infinity”
  – assume all light comes from a single point
  – which is very far away from the scene

• reasonable assumption for vision where
  one of two cases tend to hold
  – source is much smaller than the scene (e.g. a light-bulb)
  – source is very far away (e.g. the sun)

• hence, in general, relative to its size and
  the size of the scene the source can be
  considered distant
Point source at infinity

- why is this interesting?
  - because a **PS @ infinity** only emits light in one direction
  - this can be understood intuitively
  - e.g. while a nearby source hits the object in all directions

  - rays that originate far away become parallel by the time they reach the object

  - hence, **there is only one incoming direction of light**
Lambertian surfaces

• in summary, we have
  – PS @ infinity
  – Lambertian surface

• we know that
  – only paths with a few bounces, from source to object, matter
  – source light hits each object along single direction

• we can go back to our original scenario
Lambertian surfaces

- overall, we have an extremely simple relationship!

\[ P(\mathbf{P}_i) = E \rho(\mathbf{P}_0) \cos \theta \]

- the power at pixel \( P_i \) is the product of
  - source power \( E \),
  - albedo of the object at reflection point,
  - and angle between source direction and object normal

![Diagram showing light source, object, and pixel relationship](image)
Lambertian surfaces

- note that
  - if $n$ is the surface normal and $s$ the light direction
  - the two vectors have unit norm
  - then $\cos \theta = n \cdot s$ and

$$P(P_i) = E \rho(P_0) \vec{n}(P_0).\vec{s}$$
Lambertian surfaces

- note that
  - light direction $s$ is constant
  - but the surface normal $n$ and the albedo $\rho$ are functions on the object surface

$$P(P_i) = E \rho(P_0) \vec{n}(P_0) \cdot \vec{s}$$
Vision vs graphics

- this is a nice example of why vision is much harder than graphics

\[ P(P_i) = E \rho(P_0) \tilde{n}(P_0) \cdot \tilde{s} \]

- **graphics**: given \( \rho, n, \) and \( s \) compute \( P \)
- this is just a multiplication
- **vision**: given \( P \), find \( \rho, n, \) and \( s \)
- really hard problem
- note that both \( \rho \) and \( n \) depend on the pixel, so the # of unknowns is three times the # of constraints
- cannot be solved, unless we make assumptions about these functions
Vision vs graphics

• once again, your brain is stellar at doing this

  – why do we see two spheres of uniform color and not two flat objects that get darker as you move down the image?
  – requires preference for 3D objects, assumption that the spheres are smooth, that the light is at the top, that there are shadows ...
  – a lot of vision is really just checking what you know already!
Vision

• it turns out that if you make the right assumptions
  – it can be done
  – research problem, not perfect yet

![image](image)
![shading (cos θ)](shading)
![albedo (ρ)](albedo)
Multiple light sources

• finally, note that the equation is linear on \( s \)

\[
P(P_i) = E\rho(P_0)\vec{n}(P_0) \cdot \vec{s}
\]

• if we have \( n \) PS @ infinity, we can just assume that \( s = s_1 + \ldots + s_n \)

\[
P = E\rho(P_0)\vec{n}(P_0) \cdot \sum_k \vec{s}_k
\]

\[
= \sum_k E\rho(P_0)\vec{n}(P_0) \cdot \vec{s}_k = \sum_i P_i
\]

• resulting image is sum of the images due to each source
Any questions?