Radiometry

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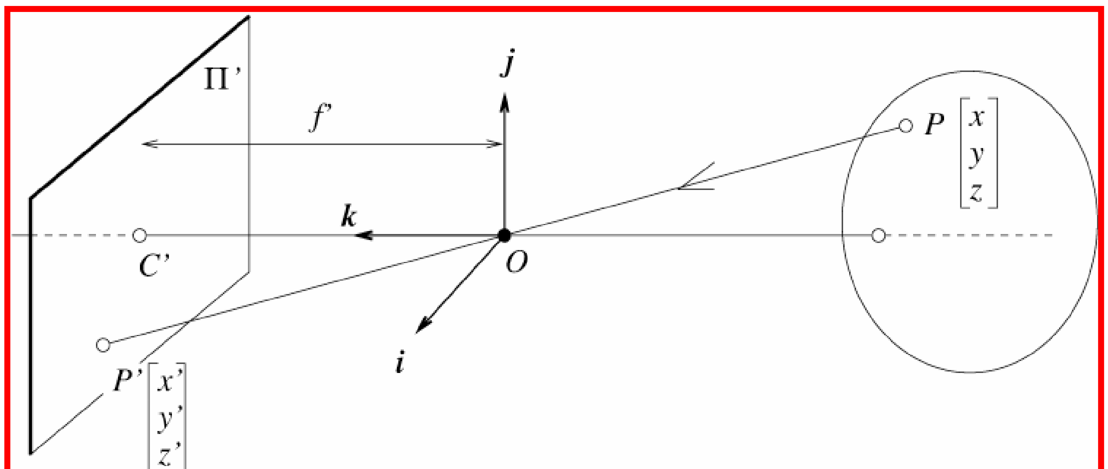
Light

- Last class: geometry of image formation
- pinhole camera:
  - point (x,y,z) in 3D scene projected into image pixel of coordinates (x', y')
  - according to the perspective projection equation:



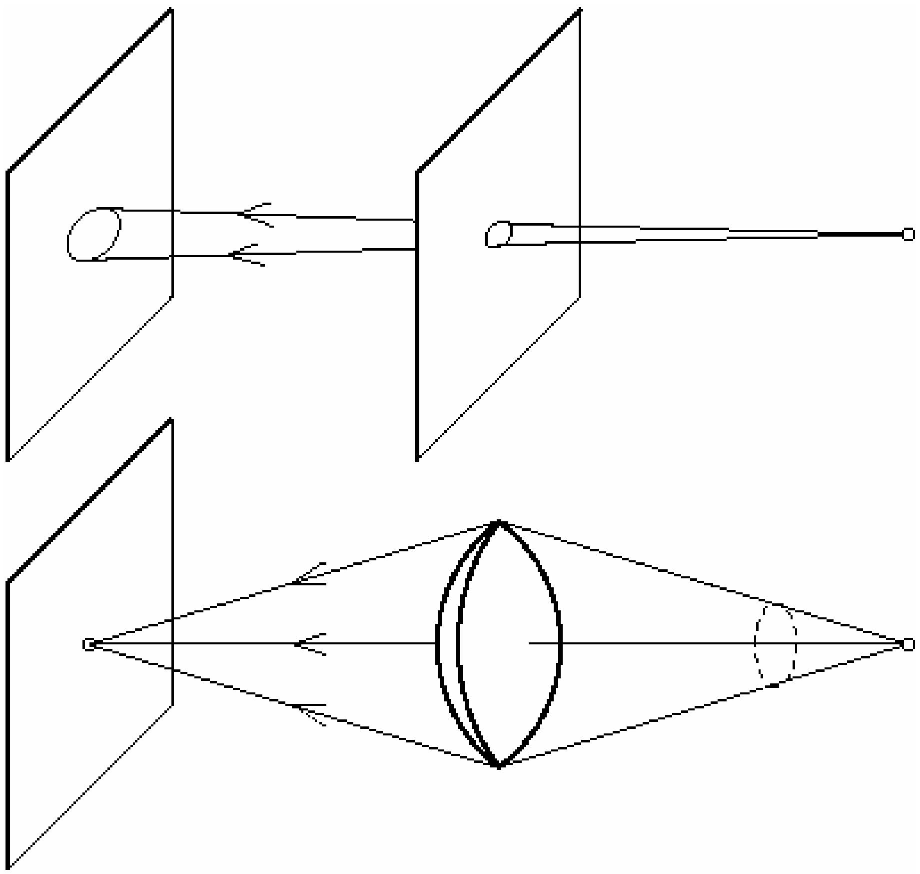
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = f \begin{pmatrix} x/z \\ y/z \end{pmatrix}$$

Perspective projection

- the inverse dependence on depth (Z)
  - causes objects to shrink with distance
- while pinhole is a good mathematical model
- in practice, cannot really use it
  - not enough light for good pictures

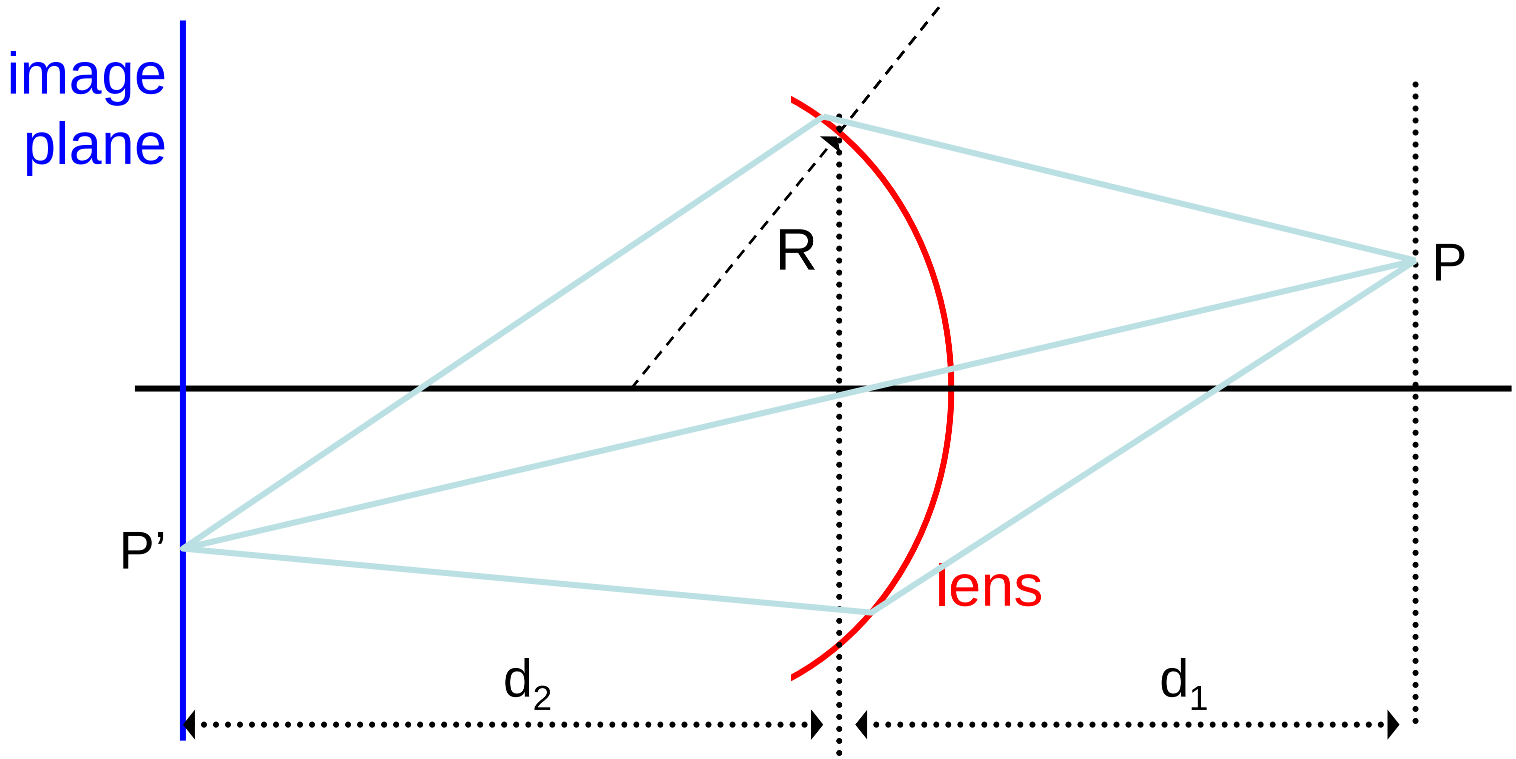
Lenses

- the basic idea is:
  - lets make the aperture bigger so that we can have many rays of light into the camera
  - to avoid blurring we need to concentrate all the rays that start in the same 3D point
  - so that they end up on the same image plane point



Lenses

- fundamental relation $$\frac{1}{d_1} \approx \frac{1}{n_1}\left(\frac{n_2 - n_1}{R} - \frac{n_2}{d_2}\right)$$
  - note that it does not depend on the vertical position of P
  - we can show that it holds for all rays that start in the plane of P



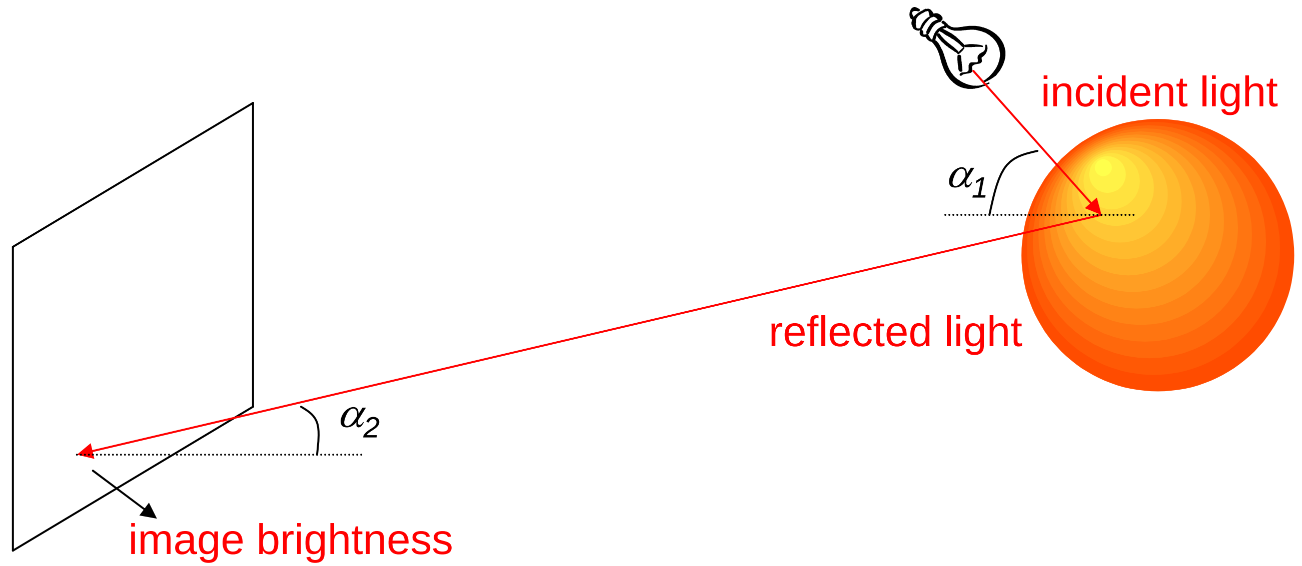
Lenses

- note that, in general,
  - can only have in focus objects that are in a certain depth range
  - this is why background is sometimes out of focus

  - by controlling the focus you are effectively changing the plane of the rays that converge on the image plane without blur
- for math simplicity, we will work with pinhole model!

Light

- today : what is the pixel brightness or image intensity?
- clearly, depends on three factors:
  - lighting of the scene
  - the reflectance properties of the material
  - various angles



Radiance

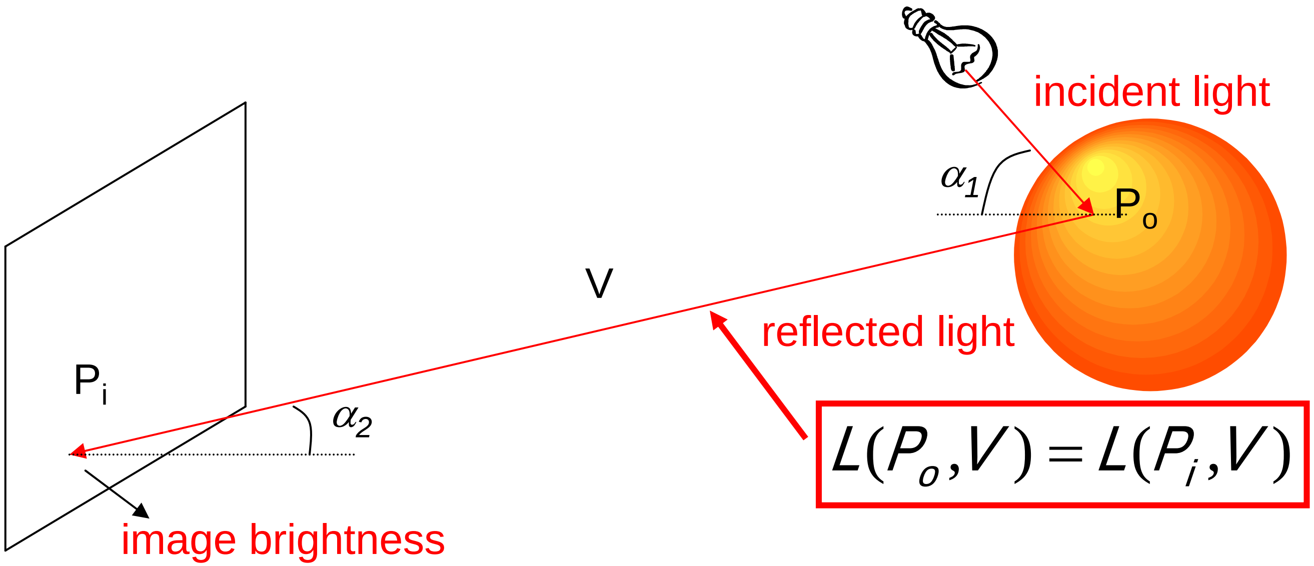
- to study light
- first important concept is radiance
  - appropriate unit for measuring distribution of light
- **Definition:** radiance is
  - power (energy/unit time) traveling at x in direction **V**
  - per unit area perpendicular to **V**
  - per unit solid angle

$L(x,V)$

- measured in
  - watts/square meter x steradian (w x m^{-2} x sr^{-1})
  - (steradian = radian squared)
- it follows that it is a function of a point x and a direction V

Radiance

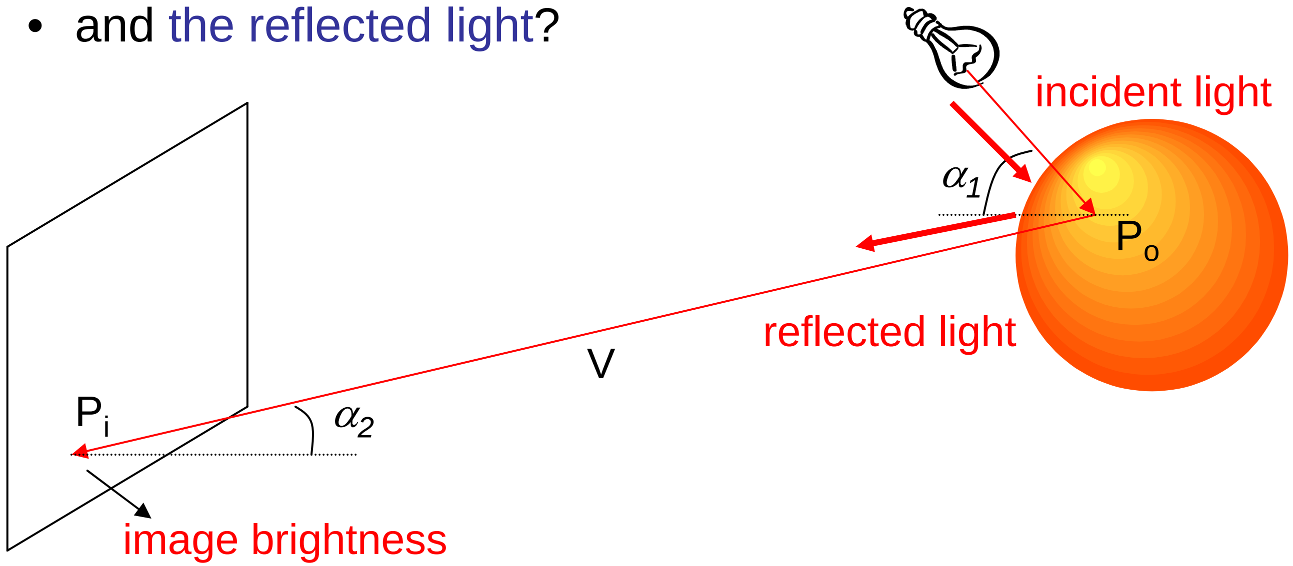
- important property
  - in a lossless medium (e.g. air)
  - whatever radiance is emitted by the object at P_o
  - is the radiance that is received by the image at P_i



- “in a lossless medium radiance is constant along straight lines”

Light

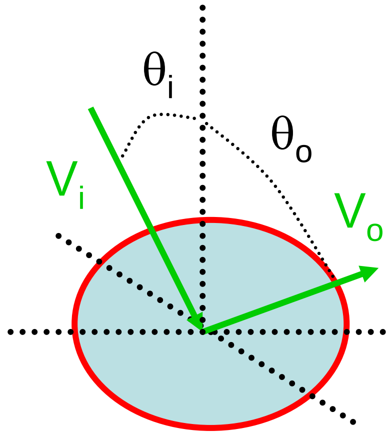
- the next question is:
  - what is the relation between
    - the illumination that reaches the object
    - and the reflected light?



- this is measured by the bidirectional reflectance distribution function (BRDF)

BRDF

- is the ratio of energy in outgoing direction (V_o) to incoming direction (V_i)

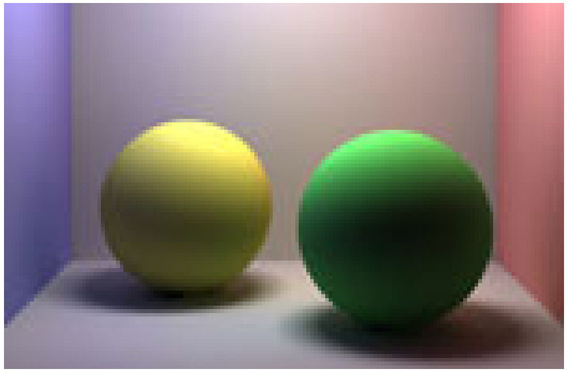


$$\rho_{bd}(P, V_i, V_o)$$

- important property (Helmoltz reciprocity)
  - BRDF is symmetric

$$\rho_{bd}(P, V_i, V_o) = \rho_{bd}(P, V_o, V_i)$$

- but we can do even simpler than this
  - for Lambertian surfaces, the BRDF does not depend direction at all
  - they reflect light equally in all directions



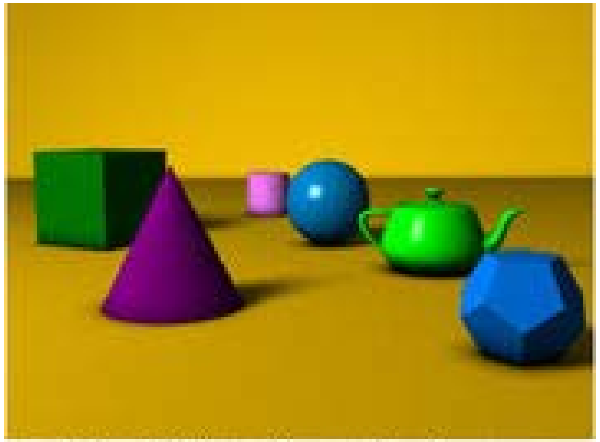
$$\rho_{bd}(P, V_i, V_o) = \rho\ (P)$$

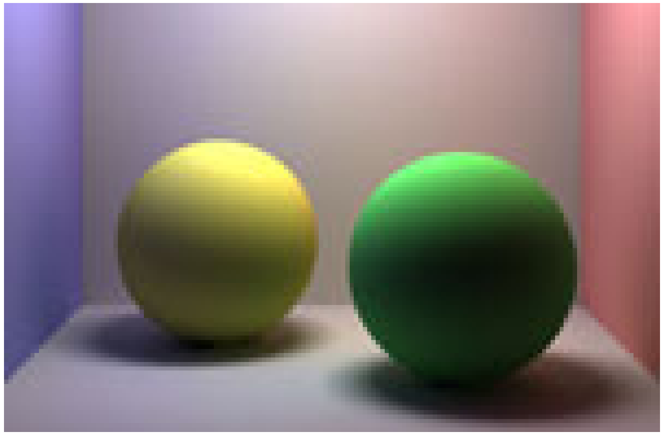
Albedo

- in this case the surface is described by its albedo

$$\rho(P)$$

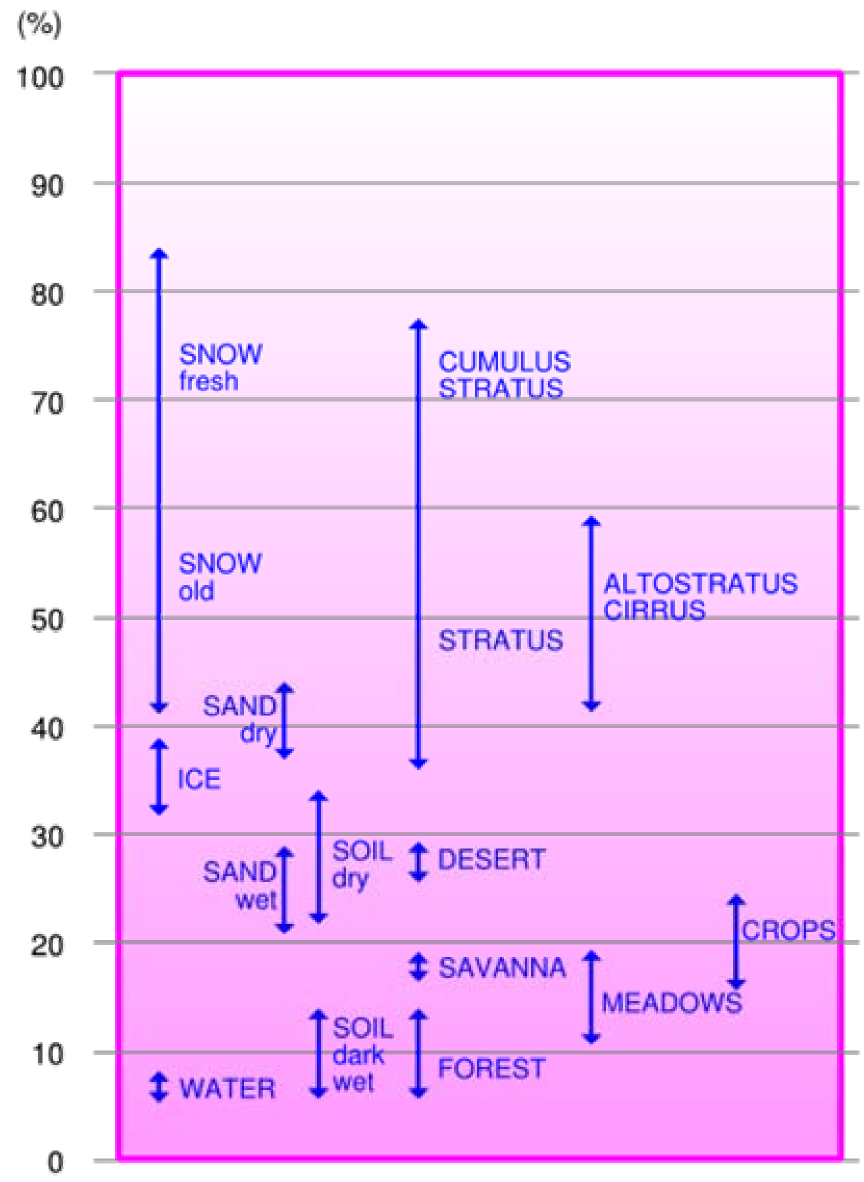
- note that
  - most surfaces are not Lambertian,
  - but the Lambertian assumption makes the equations a lot easier
  - commonly used in practice, even though Lambertian objects do not appear realistic (not good enough for graphics)





Albedo

- in laymen's terms:
  - albedo is percentage of light reflected by an object
  - it depends on the color and material properties of the object
  - light colors reflect more light (why you should wear white in the desert)
  - this turns out to have major consequences for object temperature
  - e.g., it is one of the main justifications for global warming

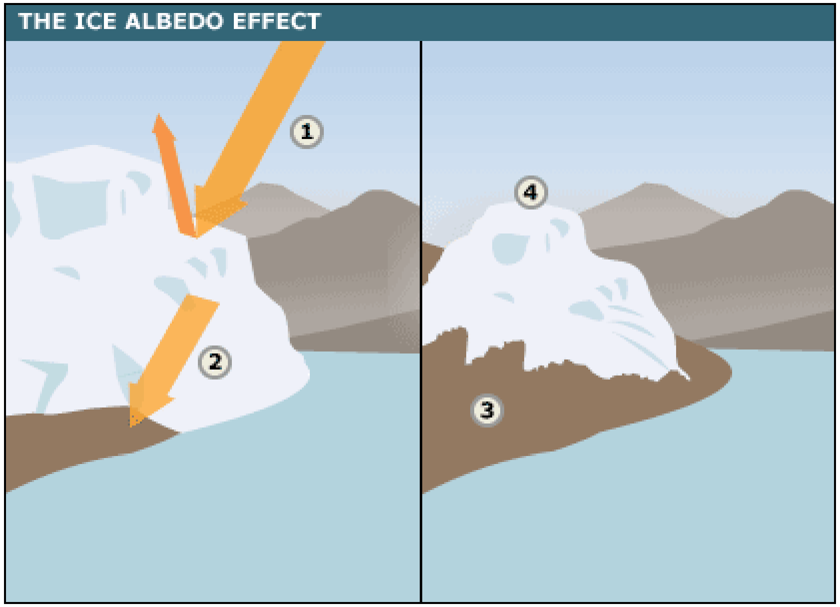


Albedo and global warming

- snow turns out to have the largest possible albedo, and reflects almost 100% of the light
- most other objects absorb light, and heat up

Albedo and global warming

- by reflecting most of the incident light
- the polar cap cools off the planet
- as ice melts, less light is reflected
- the planet warms up, more ice melts, etc.
- this is one of the main reasons for global warming



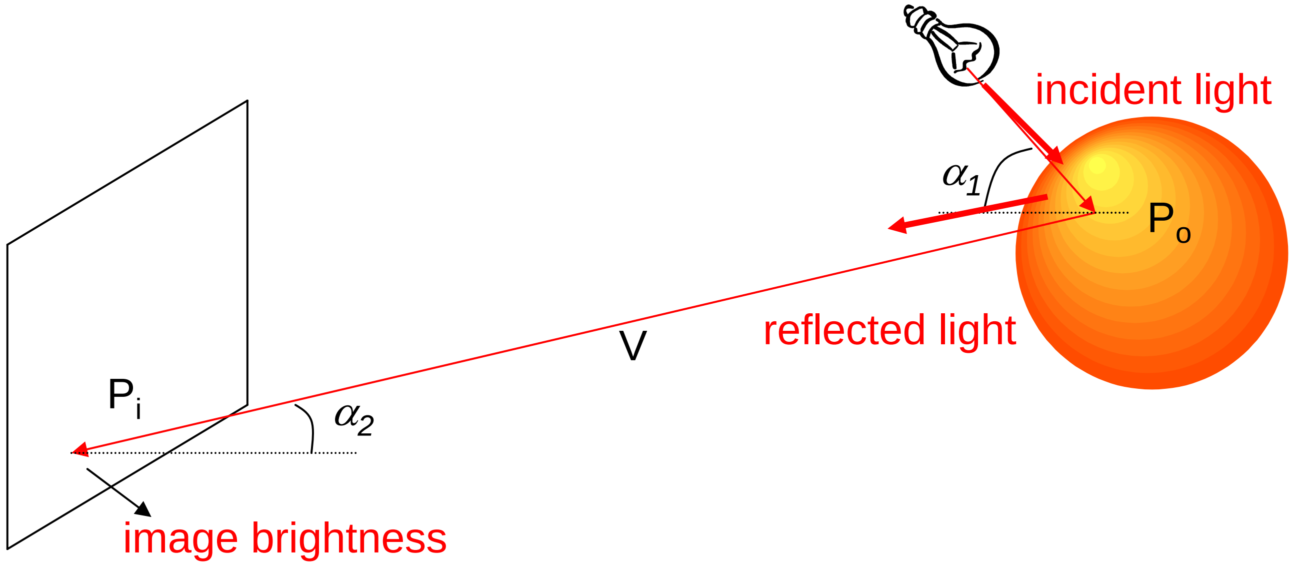
1 Light colored ice reflects back the Sun's energy efficiently.
2 Exposed land is darker colored and absorbs more energy.
3 As the ice melts, more land is exposed. This absorbs more heat, melting more ice.
4 The altitude of the melting ice is reduced so it becomes harder for new ice to form.

Albedo and parking lots

- these day, this effect is taken very seriously
- it turns out that increasing reflections is useful in many other ways
- for example, it can save a lot of lighting (energy)
- an example of how paving parking lots with pavement of higher reflectance can make a difference

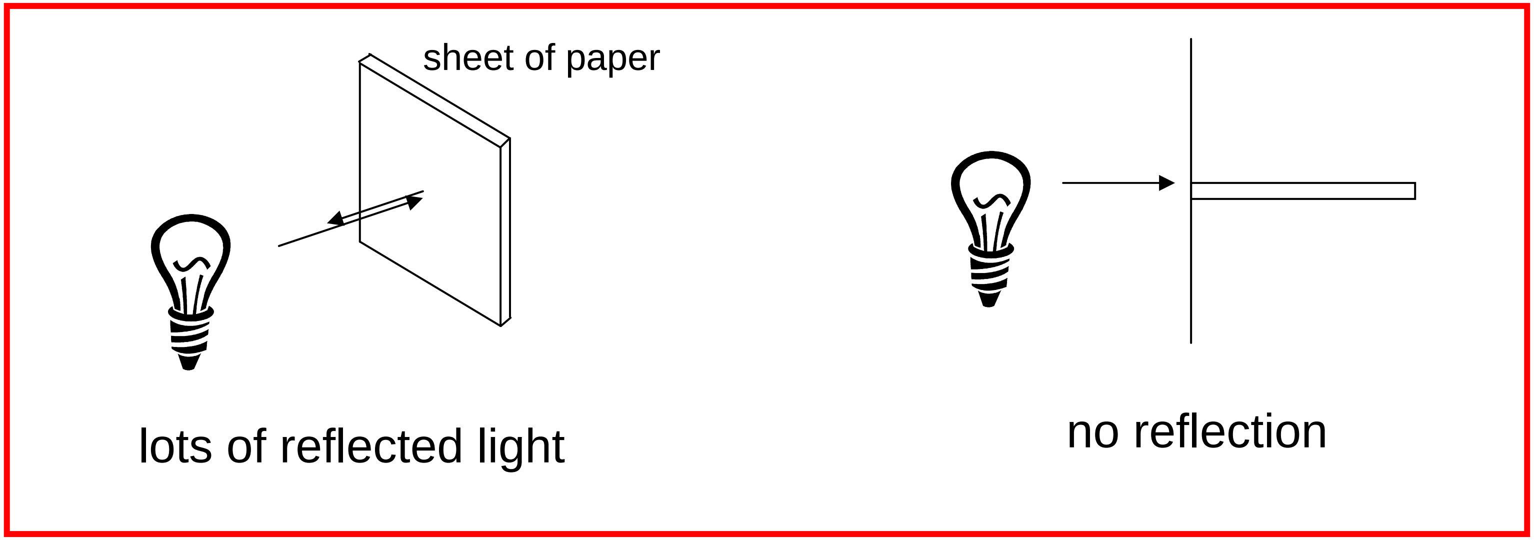
Angles

- is the object albedo the only factor that matters for how much light it reflects?
- no, the angle at which the light is incident also matters



Light

- this is easy to see
- consider the following experiment



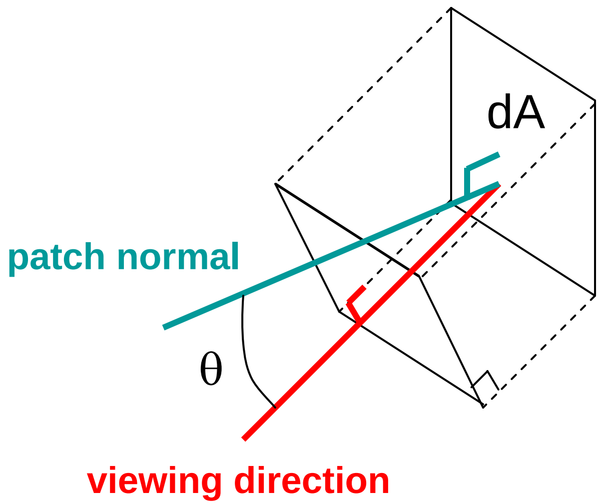
- energy absorbed by object depends on its surface area
- this varies with the incident angle
- concept that captures this dependence on angles is that of foreshortening

Light

- **foreshortening:** very important concept
  - tilted surface looks smaller than when seen at 90^o
  - best understood by example
  - if I show you a tilted person it looks smaller than when you view you at 90^o

Light

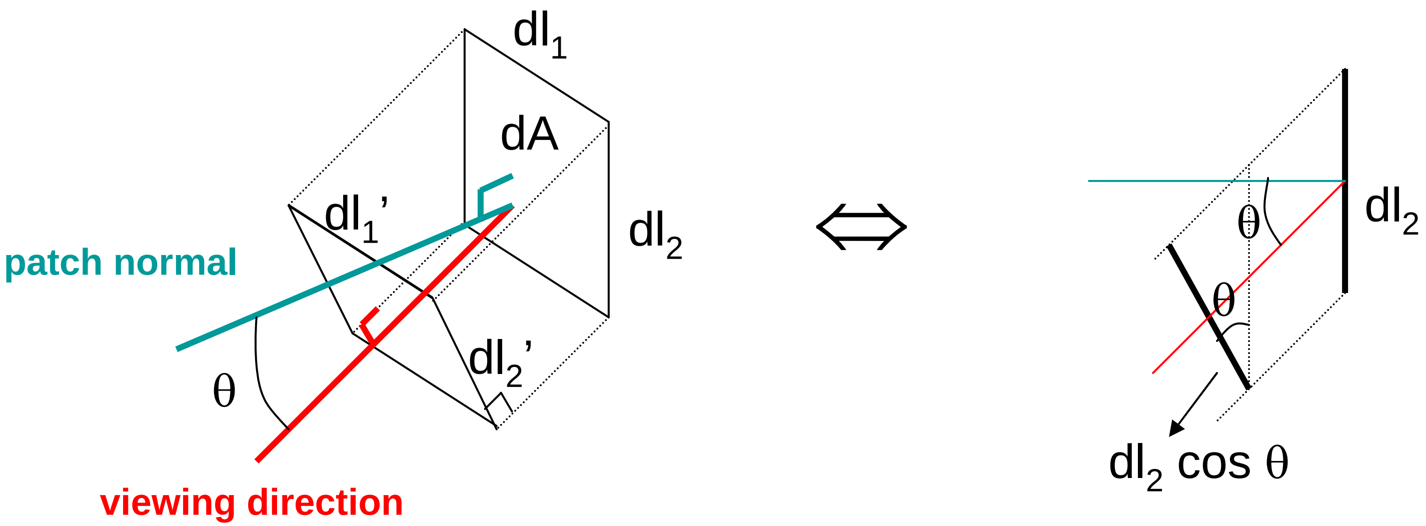
- what is the **foreshortened area** for a patch of area dA?



- it depends on the angle θ between
  - the normal to the patch
  - viewing direction
- for a known area dA we can actually compute the foreshortening factor

Light

- this is easy for a simple case

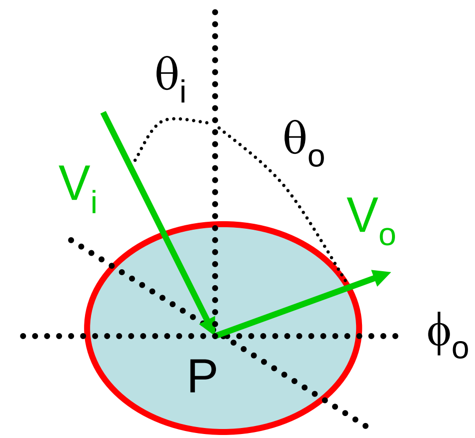


- foreshortened area is

$$dA' = dl_1' \, dl_2' = dl_1 \, dl_2 \cos \theta = dA \cos \theta$$

- it can be shown that this holds for a patch of any shape.
  - **foreshortened area = area x cos (angle between viewing direction and surface normal)**

Lambertian surfaces



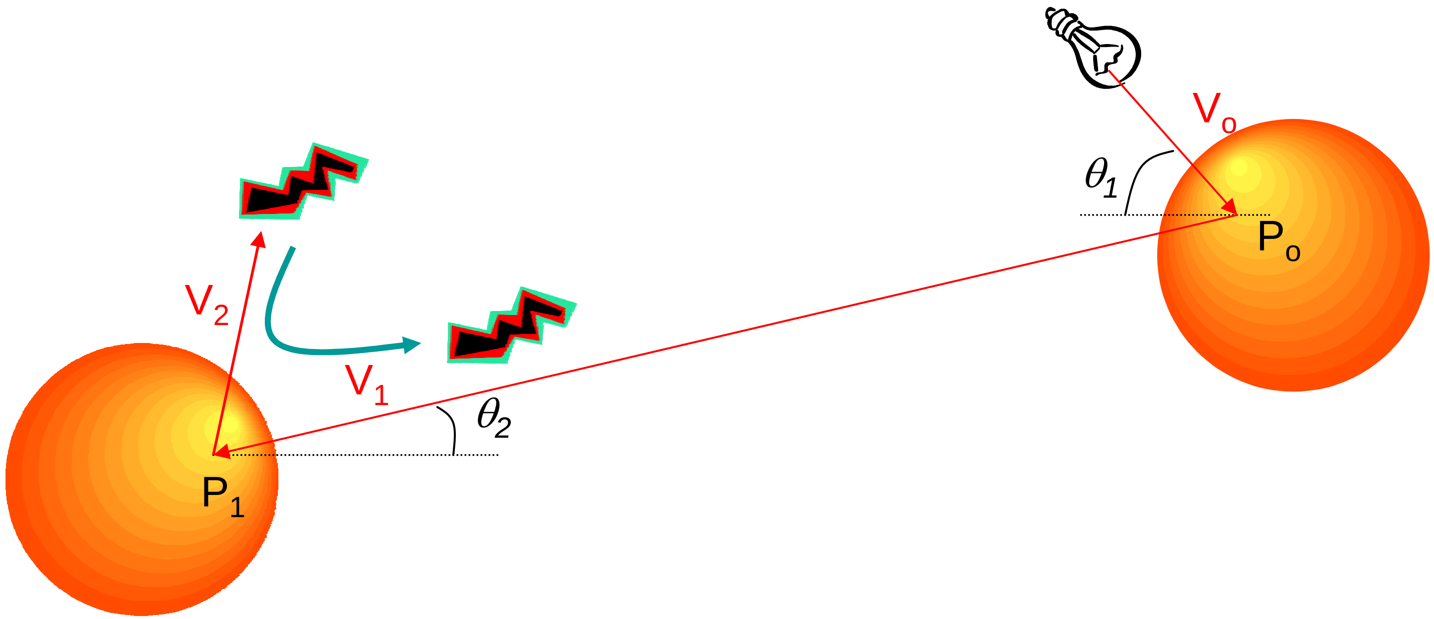
- putting everything together,
  - we have an equation for the light reflected by an object
  - assuming surface reflects equally in all directions (Lambertian)
  - outgoing radiance is

$$L(P, V_O) = \rho(P) L(P, V_i) \cos \theta_i, \forall V_o$$

  - "light reflected at point *P* in direction V_o = albedo at *P* x incident light from direction V_i x cos (normal, incident)"
  - note that
    - it holds for any outgoing direction V_o
    - is a function of V_i
    - if there are multiple incoming directions, we have to integrate over V_i

Lambertian surfaces

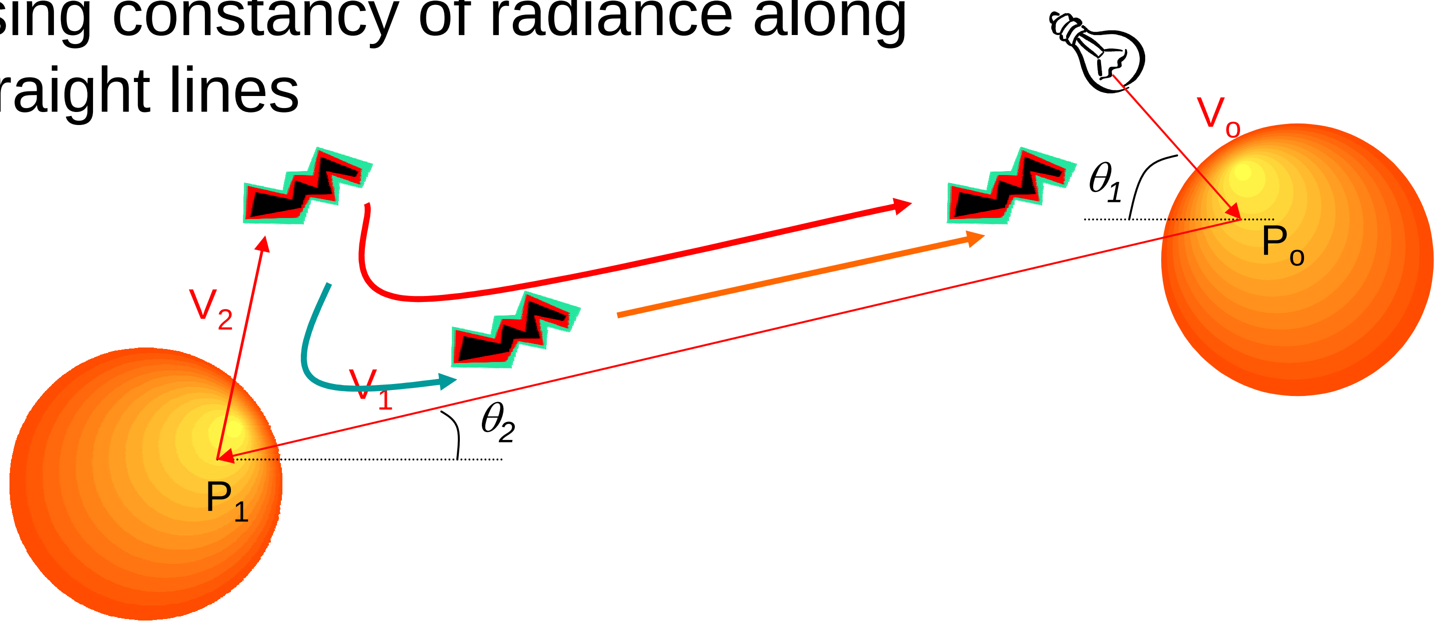
- this allows us to propagate light throughout a scene



$$L(P_1, V_2) = L(P_1, V_1)\rho(P_1)\cos\theta_2, \forall V_2$$

Lambertian surfaces

- using constancy of radiance along straight lines



$$L(P_1, V_1) = L(P_0, V_1)$$

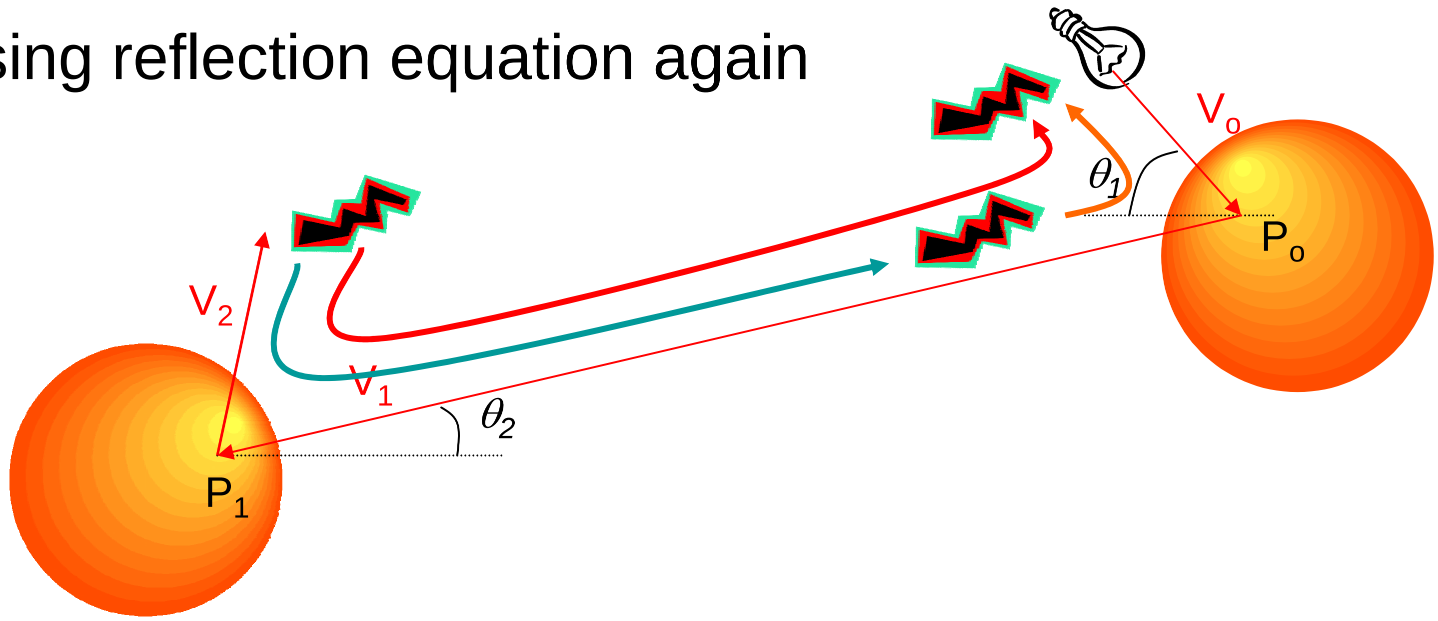
$$L(P_1, V_2) = L(P_1, V_1)\rho(P_1)\cos\theta_2 \;, \forall V_2$$

$$\Updownarrow$$

$$L(P_1, V_2) = L(P_0, V_1)\rho(P_1)\cos\theta_2 \;, \forall, V_2$$

Lambertian surfaces

- using reflection equation again



$$L(P_0,V_1) = L(P_0,V_0)\rho(P_0)\cos\theta_1, \forall V_1$$

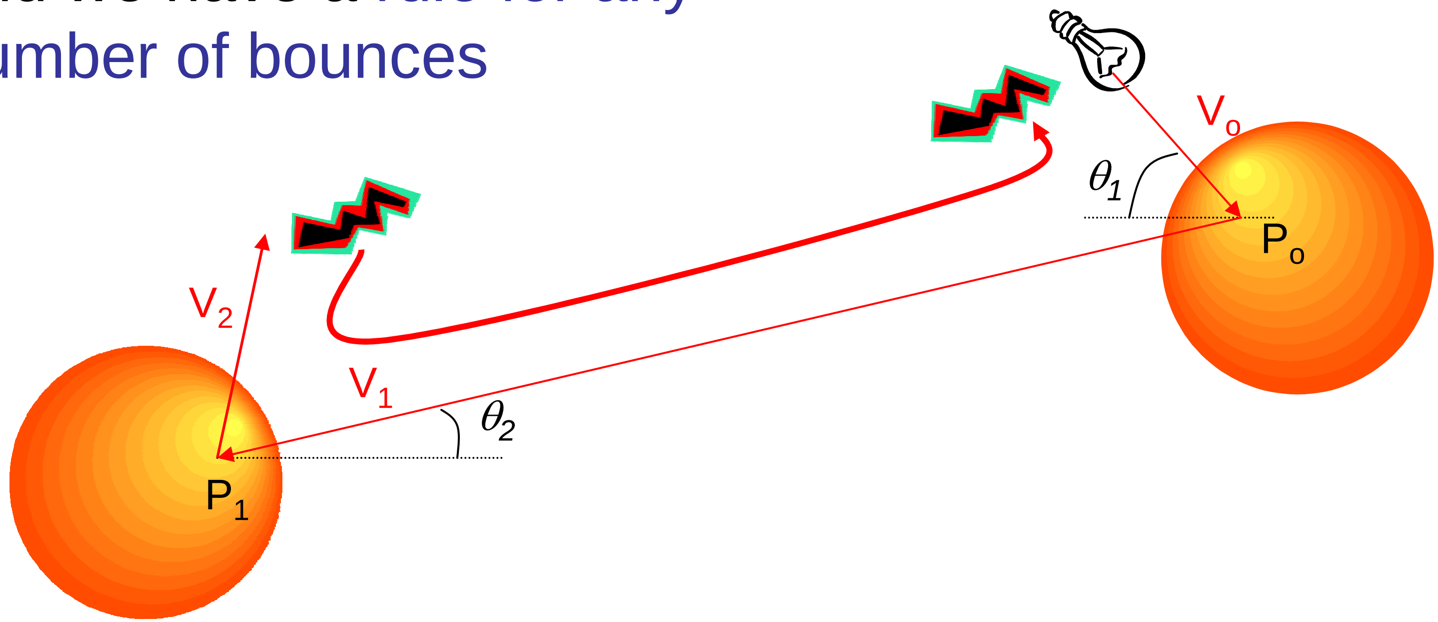
$$L(P_1,V_2) = L(P_0,V_1)\rho(P_1)\cos\theta_2, \forall V_2$$

$$\Updownarrow$$

$$L(P_1,V_2) = L(P_0,V_0)\rho(P_1)\rho(P_0)\cos\theta_2 \text{ x } \cos\theta_1$$

Lambertian surfaces

- and we have a rule for any number of bounces



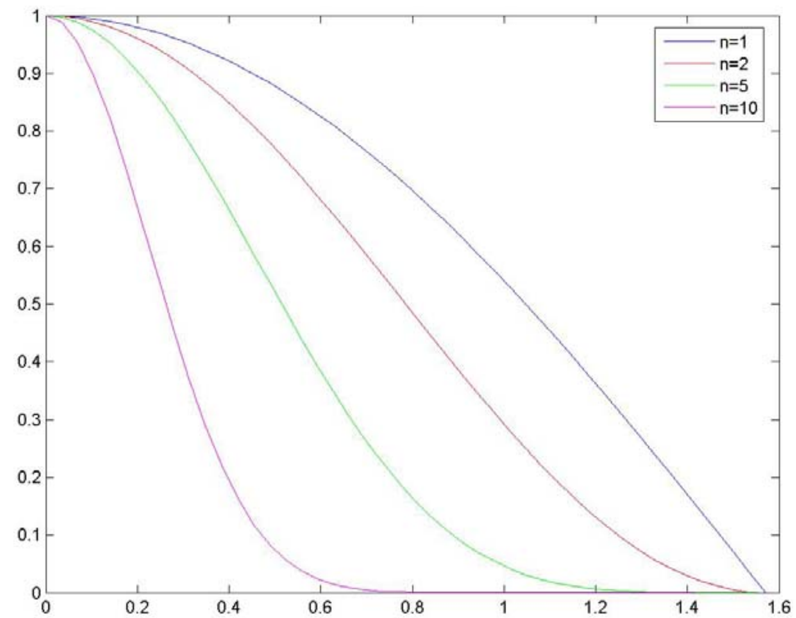
$$L(P_n, V) = L(P_0, V_0)\left[\prod_{i=0}^{n} \rho(P_i)\right]\left[\prod_{i=1}^{n+1} \cos\theta_i\right], \forall V$$

Lambertian surfaces

- note that on

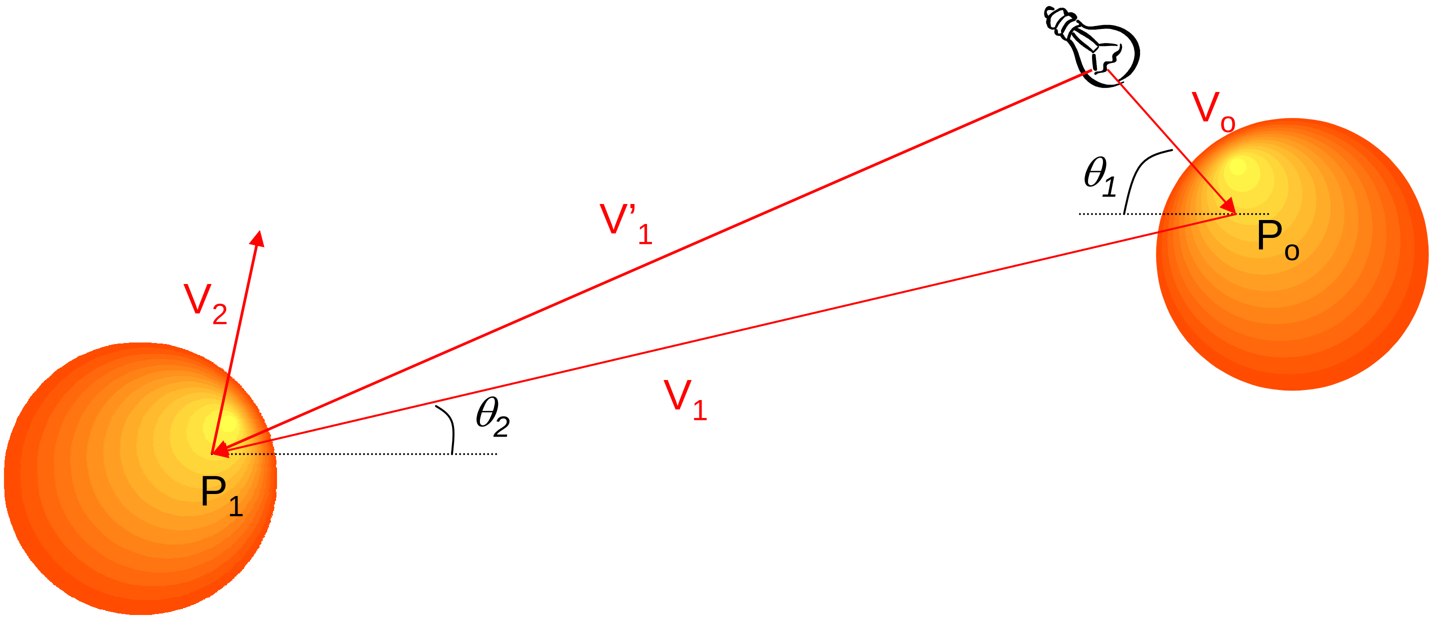
$$L(P_n, V) = L(P_0, V_0)\left[\prod_{i=0}^{n} \rho(P_i)\right]\left[\prod_{i=1}^{n+1} \cos\theta_i\right], \forall V$$

  - unless all cosines are close to 1
  - their product goes to zero quickly
  - e.g. see decay of $\cos^n(\theta)$ with n
  - this means that only light that arrives frontally to all the bounces gets propagated very far
  - such an alignment is very unlikely
  - we don't really have to worry about many bounces
  - the process becomes tractable



Lambertian surfaces

- on the other hand,
  - there are still various single-bounce paths
  - e.g. each source has a single bounce path to each non-shaded object
  - to deal with this we need to know more about light sources



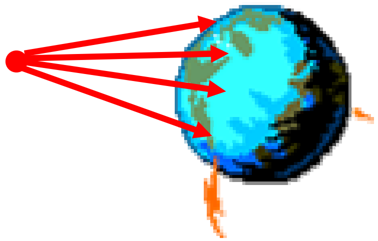
Light sources

- most common model is "point source at infinity"
  - assume all light comes from a single point
  - which is very far away from the scene

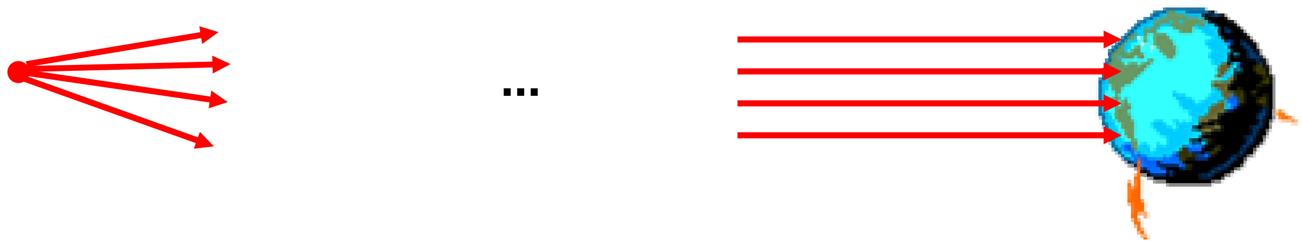
- reasonable assumption for vision where one of two cases tend to hold
  - source is much smaller than the scene (e.g. a light-bulb)
  - source is very far away (e.g. the sun)
- hence, in general, relative to its size and the size of the scene the source can be considered distant

Point source at infinity

- why is this interesting?
  - because a PS @ infinity only emits light in one direction
  - this can be understood intuitively
  - e.g. while a nearby source hits the object in all directions

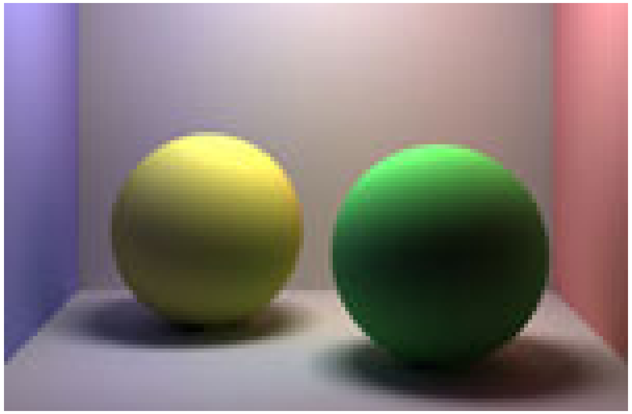


  - rays that originate far away become parallel by the time they reach the object

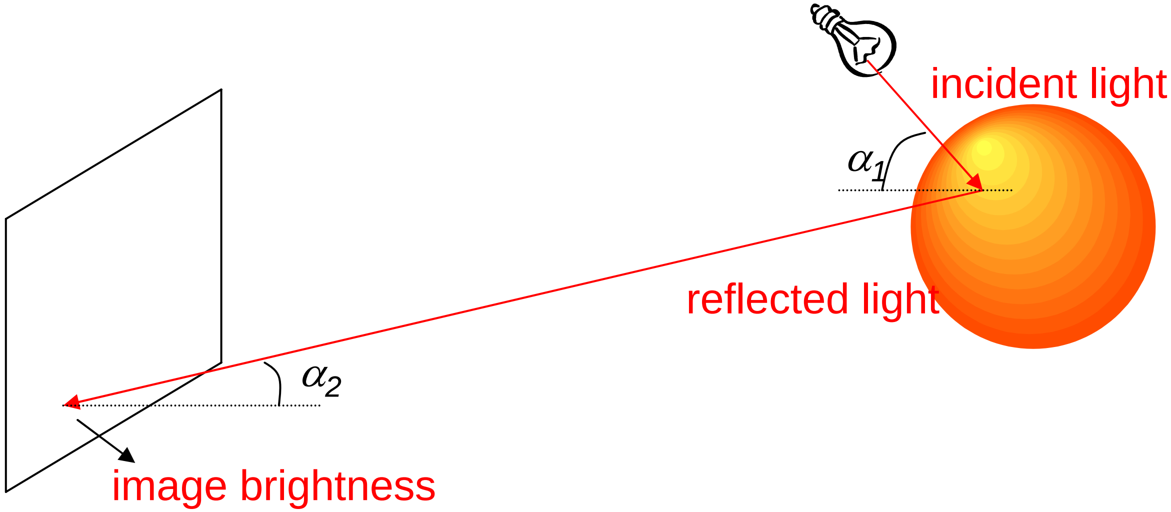


  - hence, there is only one incoming direction of light

Lambertian surfaces



- in summary, we have
  - PS @ infinity
  - Lambertian surface
- we know that
  - only paths with a few bounces, from source to object, matter
  - source light hits each object along single direction
- we can go back to our original scenario

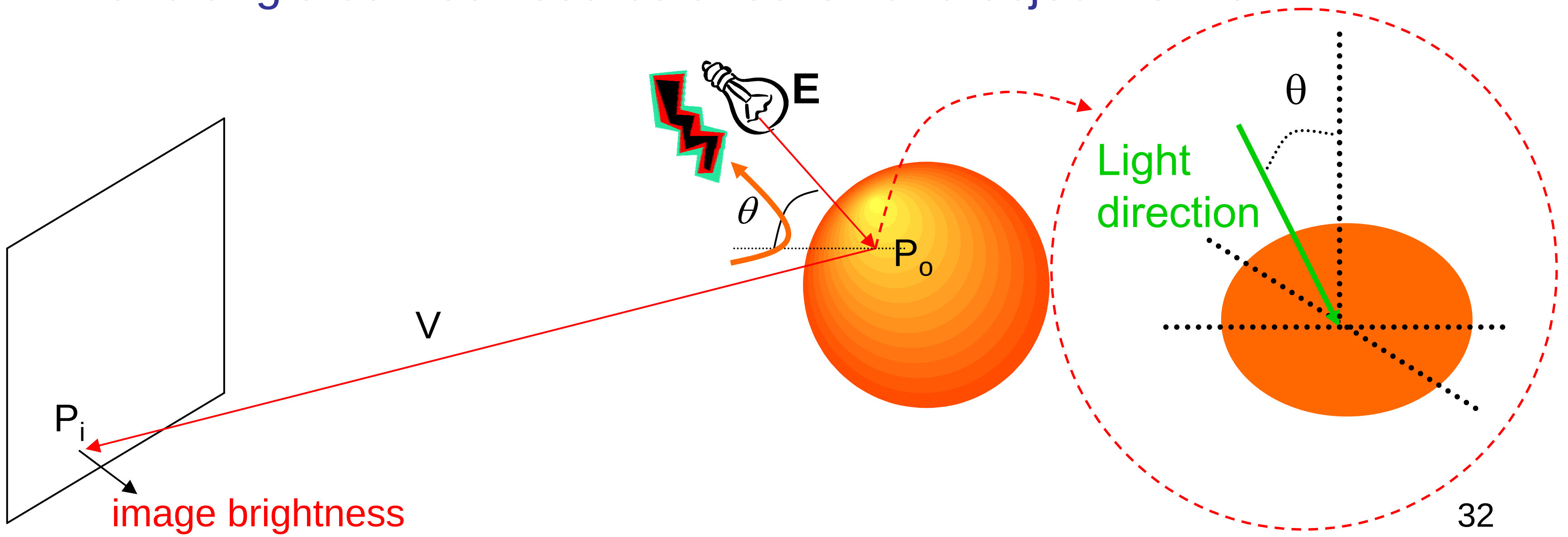


Lambertian surfaces

- overall, we have an extremely simple relationship!

$$P(P_i) = E\rho(P_0)\cos\theta$$

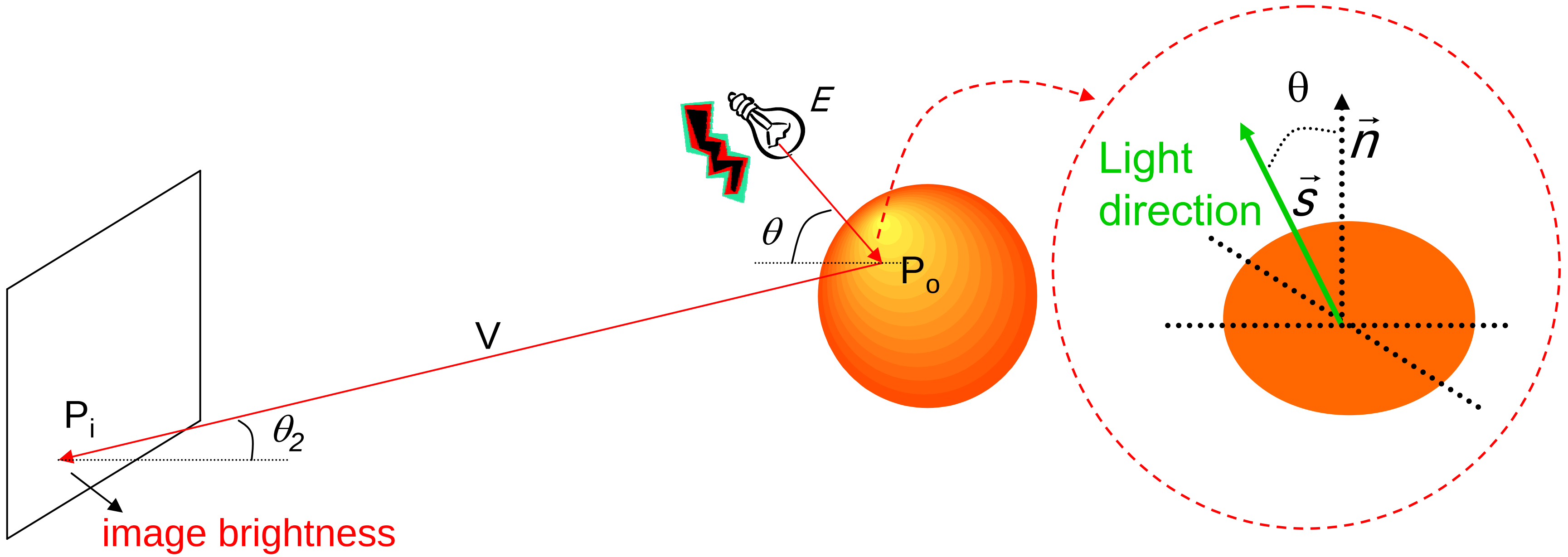
- the power at pixel P_i is the product of
  - source power E,
  - albedo of the object at reflection point,
  - and angle between source direction and object normal



Lambertian surfaces

- note that
  - if ***n*** is the surface normal and ***s*** the light direction
  - the two vectors have unit norm
  - then *cos* θ = ***n . s*** and

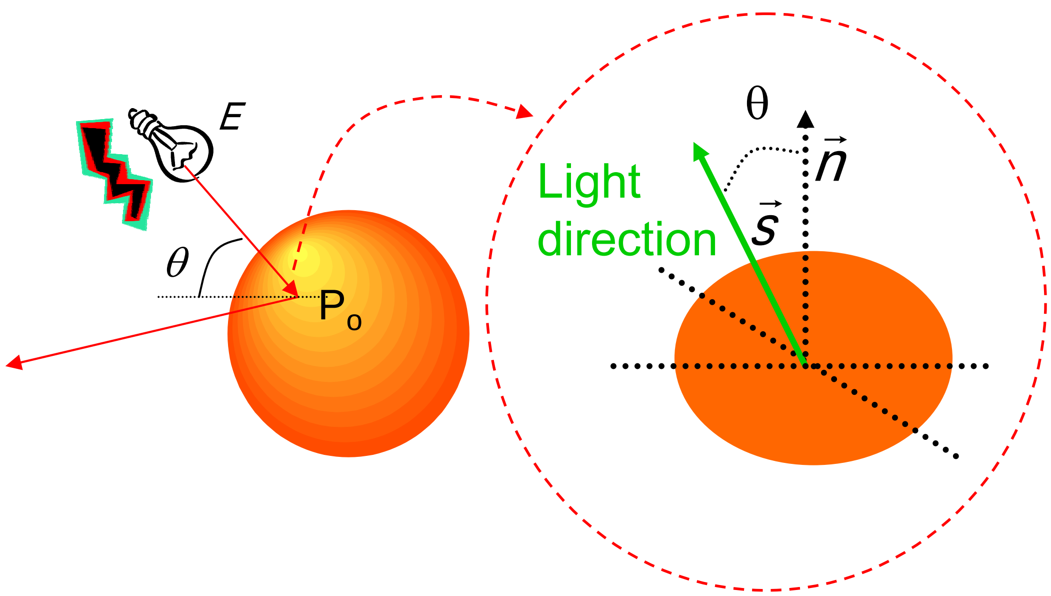
$$P(P_i) = E\rho(P_0)\vec{n}(P_0).\vec{s}$$

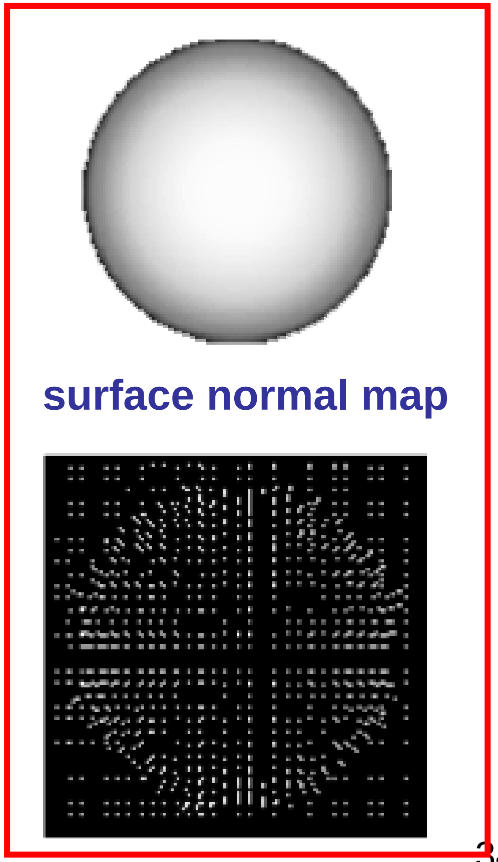


Lambertian surfaces

- note that
  - light direction ***s*** is constant
  - but the surface normal ***n*** and the albedo ρ are functions on the object surface

$$P(P_i) = E\rho(P_0)\vec{n}(P_0).\vec{s}$$



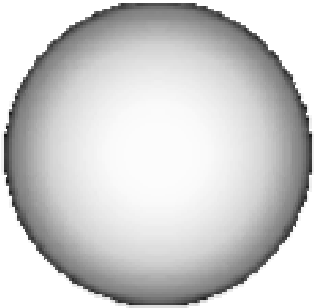


Vision vs graphics

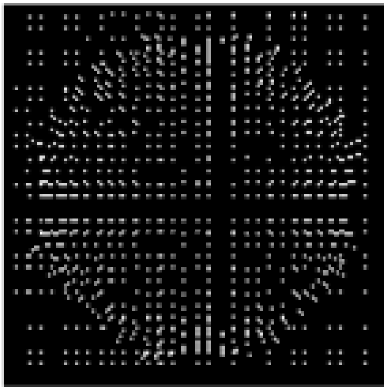
- this is a nice example of why vision is much harder than graphics

$$P(P_i) = E\rho(P_0)\vec{n}(P_0).\vec{s}$$

  - graphics: given ρ, ***n***, and ***s*** compute P
  - this is just a multiplication
  - vision: given *P*, find ρ, ***n***, and ***s***
  - really hard problem
  - note that both ρ and ***n*** depend on the pixel, so the # of unknowns is three times the # of constraints
  - cannot be solved, unless we make assumptions about these functions

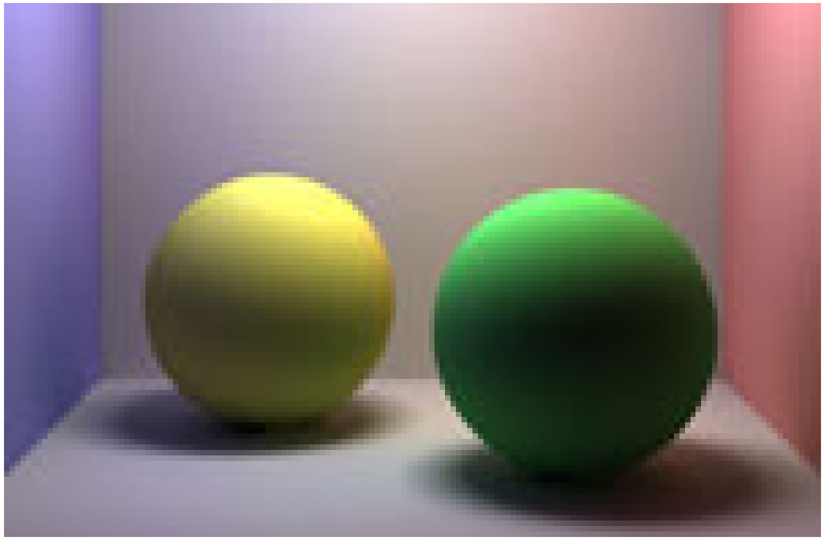


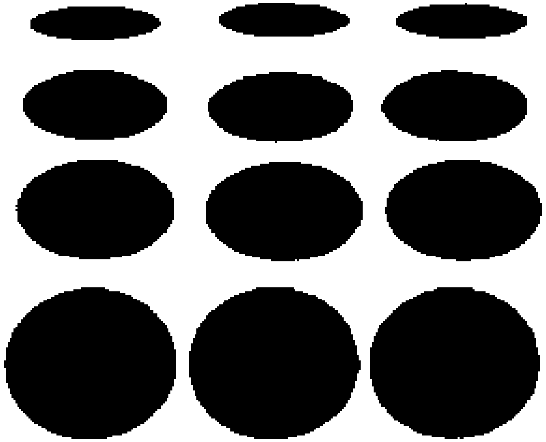
surface normal map



Vision vs graphics

- once again, your brain is stellar at doing this



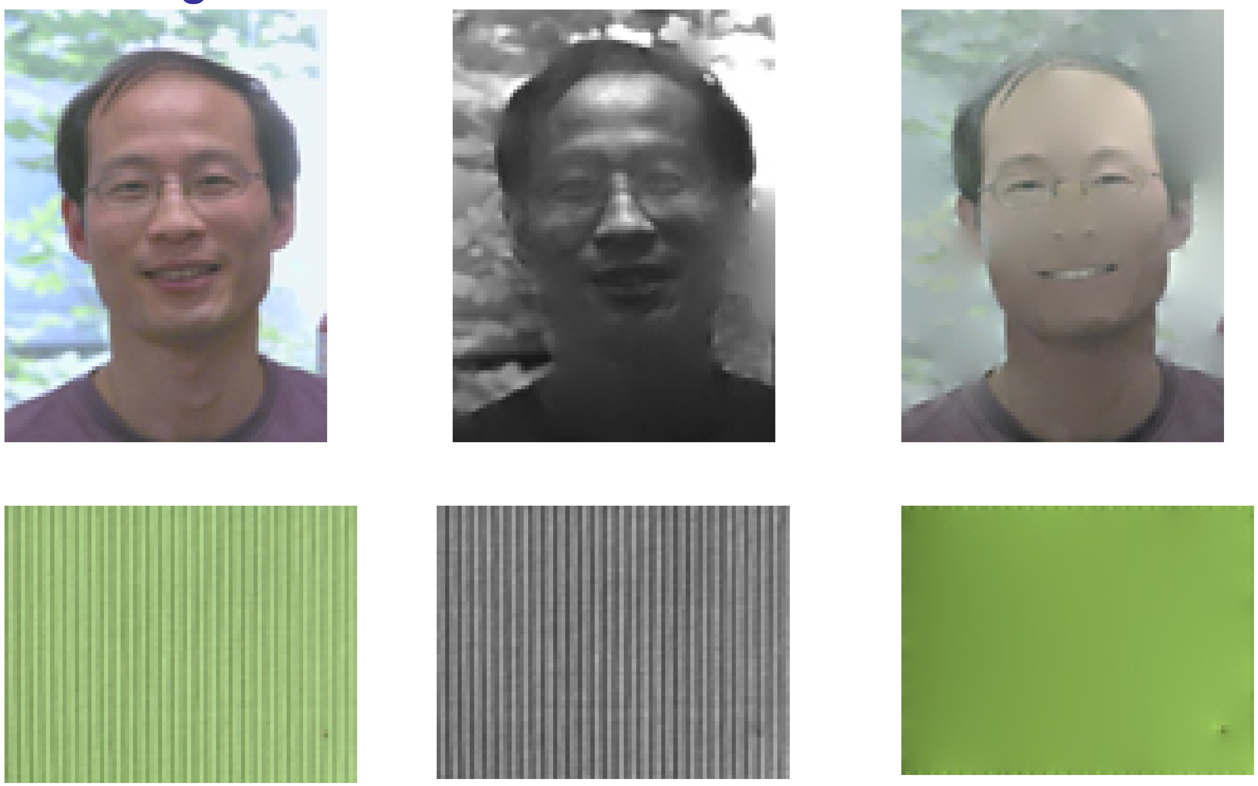


  - why do we see two spheres of uniform color and not two flat objects that get darker as you move down the image?
  - requires preference for 3D objects, assumption that the spheres are smooth, that the light is at the top, that there are shadows ...
  - a lot of vision is really just checking what you know already!

Vision

- it turns out that if you make the right assumptions
  - it can be done
  - research problem, not perfect yet

image **shading (cos θ)** **albedo (ρ)**



Multiple light sources

- finally, note that the equation is linear on **s**

$$P(P_i) = E\rho(P_0)\vec{n}(P_0).\vec{s}$$

- if we have *n* PS @ infinity, we can just assume that **s** = **s**$_1$ + …+ **s**$_n$

$$P = E\rho(P_0)\vec{n}(P_0).\sum_k \vec{s}_k$$
$$= \sum_k E\rho(P_0)\vec{n}(P_0).\vec{s}_k = \sum_i P_k$$

- resulting image is sum of the images due to each source

Any questions?