Mid-term review
ECE 161C
Electrical and Computer Engineering
University of California San Diego

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1. We have seen in class that one popular technique for edge detection is the *Canny edge detector*. It contains three parameters: a smoothing parameter σ, and two thresholds t_1 and t_2 that implement an *hysteresis* mechanism which allows control over the likelihood of starting vs following a contour. Here we will just worry with the threshold that is used to start a contour, the other one will remain constant. We denote the threshold that we allow to change by t.

Figures 1 and 2 present an image and three edge maps obtained with different parameter settings. The combinations of parameters that we used to obtain the edge maps are given by the following table.

Experiment	t	σ
1	0.2	1
2	0.2	4
3	0.4	1

For each experiment, indicate which is the resulting edge map. Provide an explanation for your assignments, i.e. why did you assign edge map x to experiment y? (Note: the assignments themselves are only worth 10% of the grade in this problem - make sure you provide a clear explanation of your thought process in determining them).

2. Consider the sequence $x(n_1, n_2)$ whose discrete space Fourier transform is

$$X(\omega_1, \omega_2) = 2 + 2\cos(w_1) - 2\cos(w_2)$$

a) Determine $x(n_1, n_2)$.

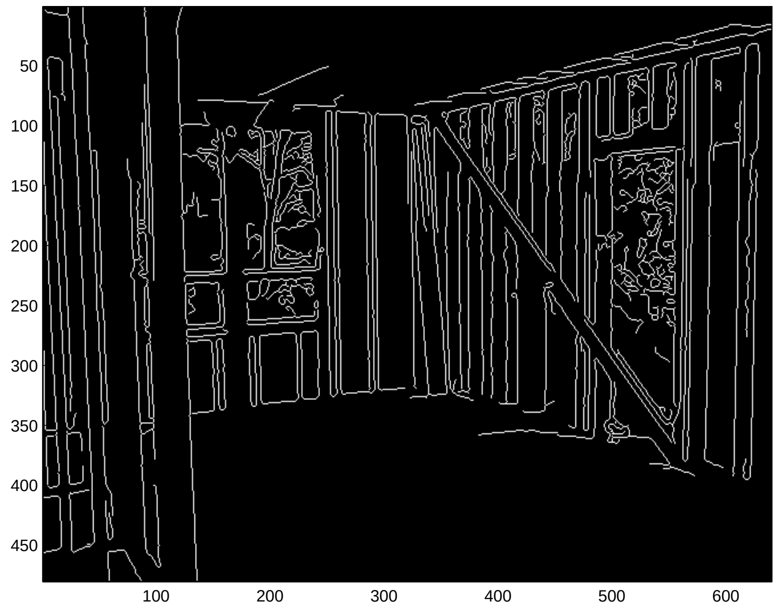
b) Is $x[n_1, n_2]$ a separable sequence? Suppose that we can add or subtract non-zero values to the (n_1, n_2) for which $x[n_1, n_2] = 0$. What is the minimum number of entries that need to be made non-zero in order to change the separability of the sequence (i.e. make it non-separable if it is separable or vice-versa)? Explain why.

c) Suppose that we can make the non-zero values of $x[n_1, n_2]$ equal to zero. What is the minimum number of entries that need to be made zero in order to change the separability of the sequence (i.e. make it non-separable if it is separable or vice-versa)?. Explain why.

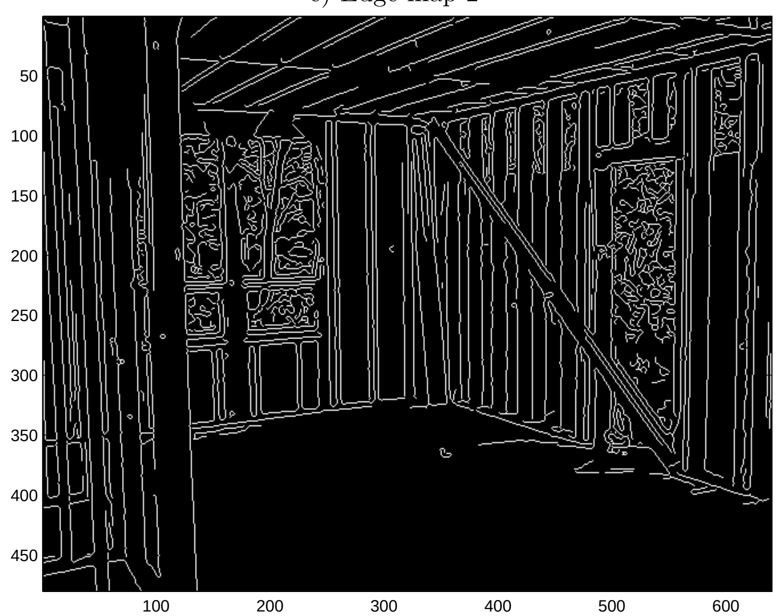
a) Image

b) Edge map 1

Figure 1:

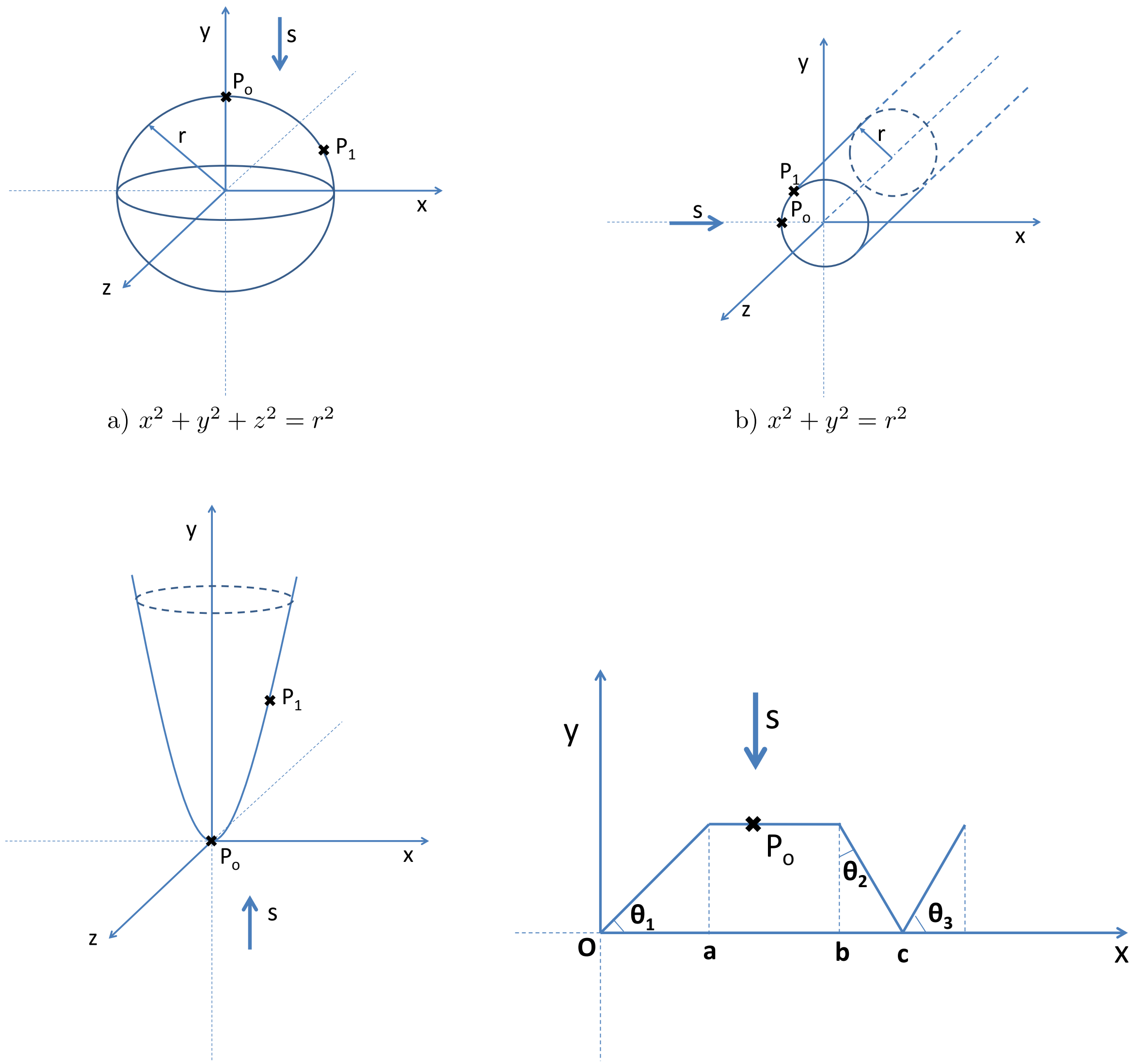


c) Edge map 2



d) Edge map 3

Figure 2:



a) $x^2 + y^2 + z^2 = r^2$

b) $x^2 + y^2 = r^2$

c) $x^2 + z^2 = y$

d) $\theta_1 = 45°, \theta_2 = 30°, \theta_3 = 60°$

Figure 3: Lambertian surfaces with uniform albedo.

3. Figures 3, a) through d), show four Lambertian surfaces with uniform albedos. Each of these surfaces is illuminated by a point source at infinity (E = 1), the direction of which is indicated by the vector s. The radiance reflected from the point P_0 on each surface is 10 units. Determine the albedo of the surfaces and derive an expression for the radiance reflected from a general point $P_1 = (x, y, z)$. Comment on the resulting illumination pattern for each case.

4. Consider the problem of correcting a poorly shot photograph, such as that shown in the left side of figure 5. The problem is that the camera was not properly oriented, and image features do not appear in their natural orientation, e.g. the buildings do not appear to be vertical. We would like to correct the image, in order to obtain something like the one shown on the right side of the figure.

We model the problem as shown below: the camera that took the picture was subject to a rotation (with respect to the "ideally aligned camera") of angle β around the optical axis. We let $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ be the coordinate system associated with the ideal camera and $\mathbf{i}'$, $\mathbf{j}'$, $\mathbf{k}'$ the one associated with the real camera.

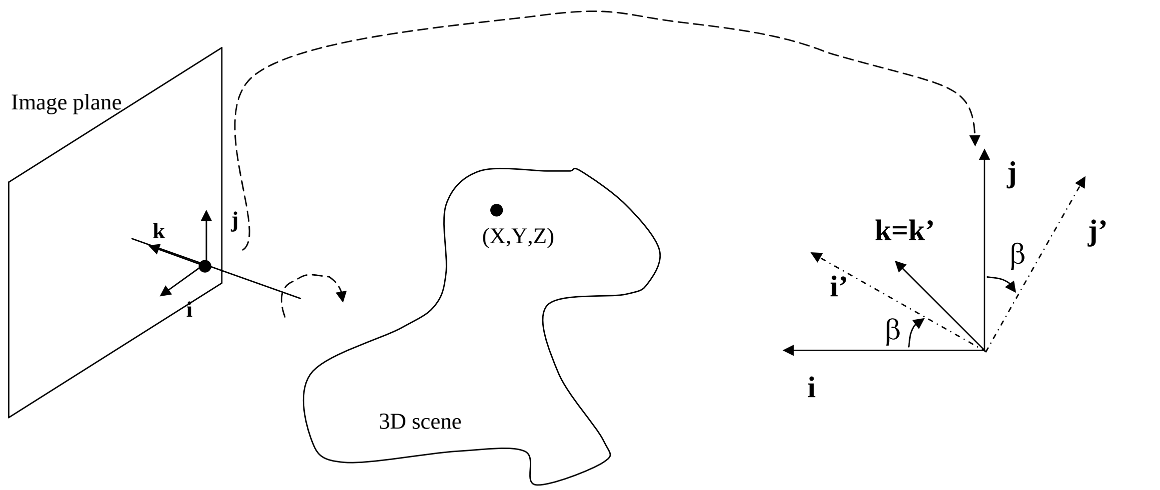


Figure 4: Left: imaging set-up for problem **4.** Right: the coordinate system of the real camera $(\mathbf{i}', \mathbf{j}', \mathbf{k}')$, is a rotation of the coordinate system of the ideal camera $(\mathbf{i}, \mathbf{j}, \mathbf{k})$.

a) Show that the coordinate systems are related by

$$\begin{aligned} \mathbf{i} &= \cos(\beta)\,\mathbf{i}' - \sin(\beta)\,\mathbf{j}' \\ \mathbf{j} &= \sin(\beta)\,\mathbf{i}' + \cos(\beta)\,\mathbf{j}' \\ \mathbf{k} &= \mathbf{k}' \end{aligned}$$

b) Consider a point of coordinates (X, Y, Z) in the old coordinate system. Using **a)**, write down the coordinates (X', Y', Z') of the point in the new coordinate system as a function of (X, Y, Z). (Note: if you find yourself inverting the system of equations in **a)** you are probably going in the wrong direction.)

c) Let (x, y) be the image coordinates of the perspective projection of the point (X, Y, Z) for the old camera, and (x', y') the image coordinates of the perspective projection of (X', Y', Z') under the new one. Derive the equation that determines (x', y') as a function of (x, y).

d) Using the equation derived in **c)** it is possible to determine the impact of the transformation on the distance between any two image points. Consider two points $\mathbf{a} = (x_1, y_1)$ and $\mathbf{b} = (x_2, y_2)$, and the points $\mathbf{a}'$ and $\mathbf{b}'$ into which they are mapped. What is the relationship between $||\mathbf{a} - \mathbf{b}||^2$ and $||\mathbf{a}' - \mathbf{b}'||^2$? (Note: here we are talking about the Euclidean distance, i.e. for $\mathbf{a} = (x, y)$

$$||\mathbf{a}||^2 = \mathbf{a}^T\mathbf{a} = x^2 + y^2.)$$

Figure 5: Left: poorly shot photograph. Right: desired image.