Homework Set Two
ECE 175
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1. Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 5 & b \\ b & 5 \end{bmatrix}$$

and the function

$$f(\mathbf{x}) = \mathbf{x}^T \mathbf{A} \mathbf{x}$$

in the range $-100 \leq x_1 \leq 100$, $-100 \leq x_2 \leq 100$.

a) using MATLAB, make both a surface and a contour plot of the function $f(\mathbf{x})$ for the three following values of b

$$b \in \{1, 5, 10\}.$$

Hand in the six plots

b) what are the eigenvalues of $\mathbf{A}$ for the three values of b?

c) how would you characterize the positive-definiteness of $\mathbf{A}$ for each b. That is, for each b, is $\mathbf{A}$ positive-definite, positive semi-definite, or neither?

d) if you were given the three words "bowl", "saddle", and "one-dimensional" to describe the function $f(\mathbf{x})$, how would you associate these descriptions to the three possible matrix classes (positive-definite, positive semi-definite, or neither)?

e) in which of the three cases is $f(\mathbf{x})$ a norm? Why?

2. a) Using the MATLAB `randn` command, generate 500 points from a 2D Gaussian of zero mean and identity covariance. Hand in a plot with the points and the contours of the Mahalanobis distance defined by this covariance.

b) Apply the transformation

$$\mathbf{T} = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}.$$

to the points. What is the new covariance matrix? How does the associated Mahalanobis distance differ from that of **a)**? Hand in a plot of the transformed data and the contours of the new Mahalanobis distance.

c) suppose that we want to transform the data of **a)** so that the new Mahalanobis distance is

$$\mathbf{x}^T \mathbf{A} \mathbf{x}$$

with

$$\mathbf{A} = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}.$$

What transformation should we apply to the data? Hand in a plot of the transformed data and the contours of the new Mahalanobis distance.

(Computer) This is the first in the series of the Computer Problems that we will do to get a feel of Machine Learning algorithms, by their application to real-world problems. We will be working with dataset of hand-written numerals in a supervised fashion. Thus the classes for our classification problem are $\mathcal{C} = \{0, 1, \ldots, 9\}$. The goal is to classify the test images into these classes such that the test error rates are minimum.

We shall use the nearest neighbor classification for the current assignment. Our database consists of a set of 5000 training images and 500 test images of hand-written numerals, where each image is 28×28 pixels with intensity values ranging from 0–255. To simplify the experiment, we shall answer the question of *Which feature to use?* by assuming the feature space to be $\mathbb{R}^{28\times 28}$ dimensional, i.e the observation or feature X are all the pixel intensity values of the image. The file `data.mat` consists of two matrices - `imageTrain` which is $28 \times 28 \times 5000$ dimensional (the third dimension indexes the different images) and `imageTest` which is $28 \times 28 \times 500$ dimensional. The file `label.mat` contains the ground truth label vectors `labelTrain` and `labelTest` for the training and testing database.

1. Use the Training images and the ground truth information to train a Nearest Neighbor Classifier using Euclidean distance metric and test it on the test database.

$$d(X, Y) = ||X - Y|| = \sqrt{\sum_{l=allpixels} |X(l) - Y(l)|^2} \tag{1}$$

   Compute and plot the error rates for each class, i.e. calculate P(Error|Class = i) for all $i = 0 \ldots 9$.

2. Calculate the total error rate, i.e. compute P(Error).

3. For 5 test images which are misclassified by the NN Classifier, display both the test image and the image from the training database which is closest to the test image. Can you explain, looking at the test image and the corresponding nearest image, why did the NN classifier failed to perform?

- To display the i^{th} image in matrix *image* use the command `imshow(`*image*`(:,:,i))`;
- To save an image you can use the command `imwrite(`*image*`(:,:,i),`*filename*`,'png');`
- It is a good idea to avoid loops as far as possible to increase the computation efficiency of matlab.